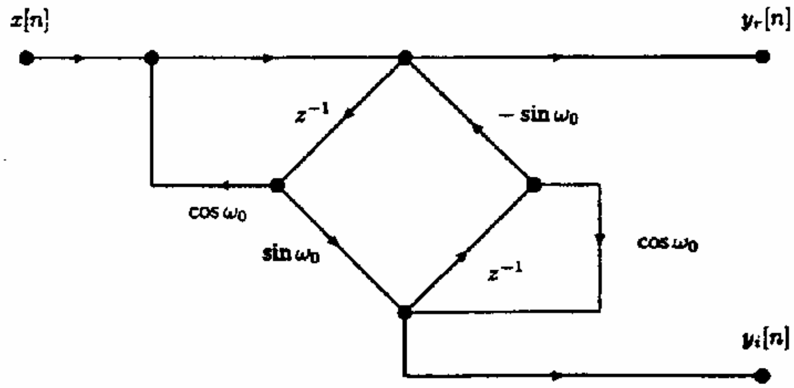
6.21.

$$h[n] = e^{j\omega_0 n}u[n] \longleftrightarrow H(z) = \frac{1}{1 - e^{j\omega_0}z^{-1}} = \frac{Y(z)}{X(z)}.$$

So $y[n] = e^{j\omega_0}y[n-1] + x[n]$. Let $y[n] = y_r[n] + jy_i[n]$. Then $y_r[n] + jy_i[n] = (\cos\omega_0 + j\sin\omega_0)(y_r[n-1] + jy_i[n-1]) + x[n]$. Separate the real and imaginary parts:

$$\begin{aligned} y_r[n] &= x[n] + \cos\omega_0 y_r[n-1] - \sin\omega_0 y_i[n-1] \\ y_i[n] &= \sin\omega_0 y_r[n-1] + \cos\omega_0 y_i[n-1]. \end{aligned}$$



6.23. Causal LTI system with system function:

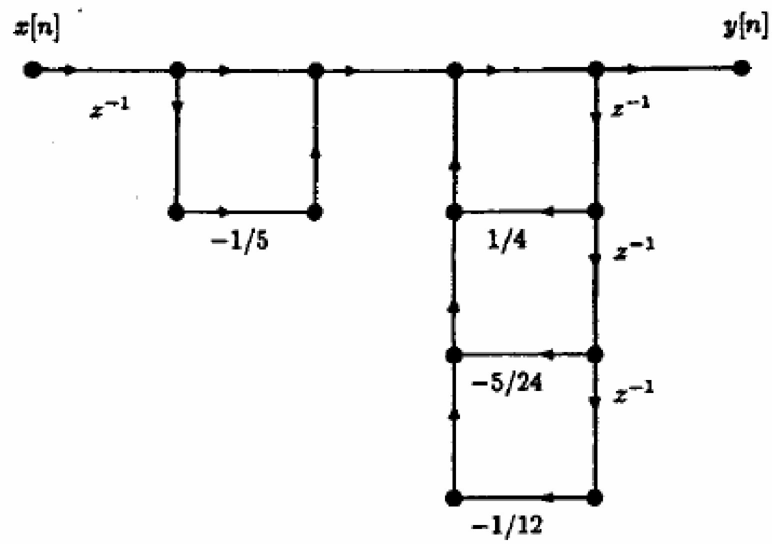
$$H(z) = \frac{1 - \frac{1}{5}z^{-1}}{(1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2})(1 + \frac{1}{4}z^{-1})}.$$

(a) (i) Direct form I.

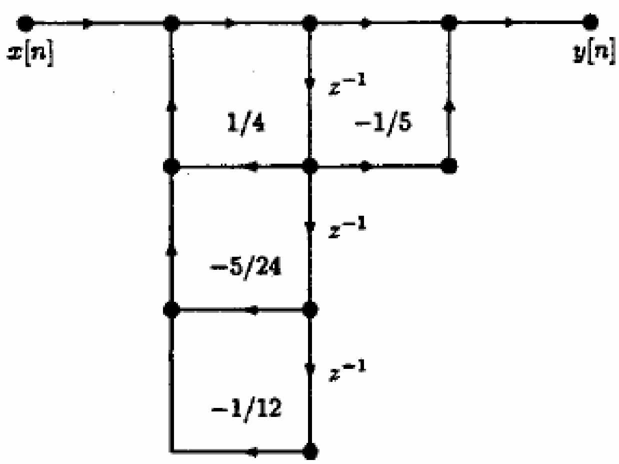
$$H(z) = \frac{1 - \frac{1}{5}z^{-1}}{1 - \frac{1}{4}z^{-1} + \frac{5}{24}z^{-2} + \frac{1}{12}z^{(-3)}}$$

so

$$b_0 = 1\ ,\ b_1 = -\frac{1}{5} \text{ and } a_1 = \frac{1}{4}\ ,\ a_2 = -\frac{5}{24}\ ,\ a_3 = -\frac{1}{12}.$$



(ii) Direct form II.

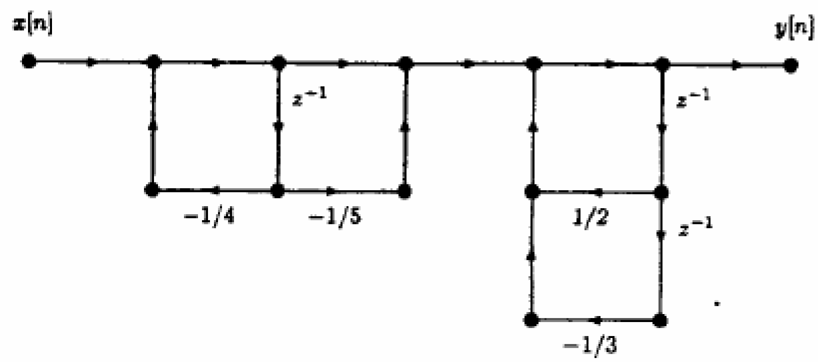


(iii) Cascade form using first and second order direct form II sections.

$$H(z) = \left(\frac{1 - \frac{1}{5}z^{-1}}{1 + \frac{1}{4}z^{-1}}\right)\left(\frac{1}{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}}\right).$$

So

$$b_{01} = 1\ ,\ b_{11} = -\tfrac{1}{5}\ ,\ b_{21} = 0\ ,$$
$$b_{02} = 1\ ,\ b_{12} = 0\ ,\ b_{22} = 0 \text{ and}$$
$$a_{11} = -\tfrac{1}{4}\ ,\ a_{21} = 0\ ,\ a_{12} = \tfrac{1}{2}\ ,\ a_{22} = -\tfrac{1}{3}.$$

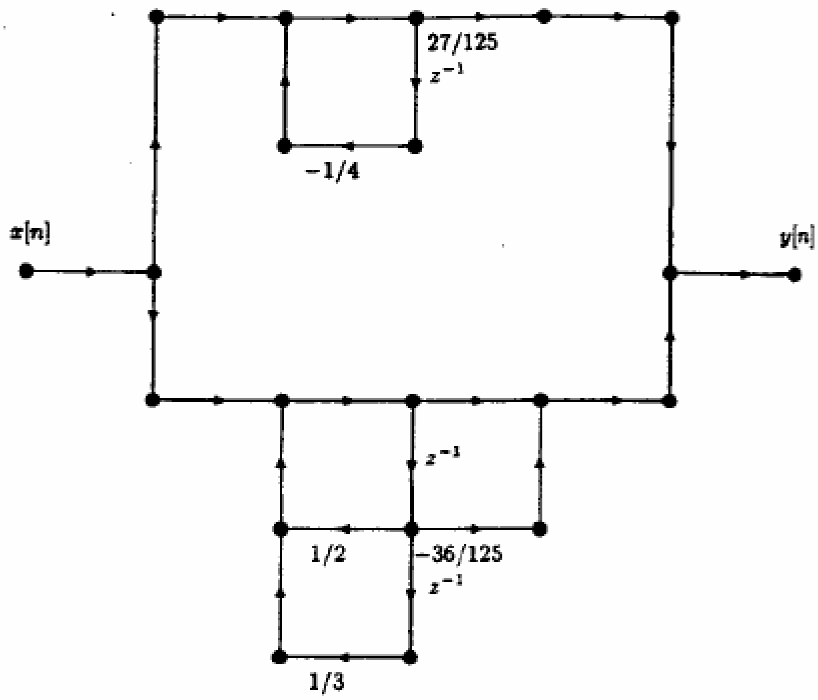


(iv) Parallel form using first and second order direct form II sections.
We can rewrite the transfer function as:

$$H(z) = \frac{\frac{27}{125}}{1 + \frac{1}{4}z^{-1}} + \frac{\frac{98}{125} - \frac{36}{125}z^{-1}}{1 - \frac{1}{2}z^{-1} - \frac{1}{3}z^{-2}}.$$

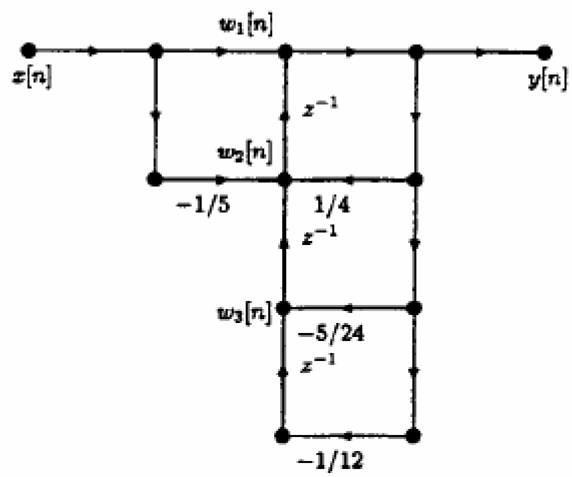
So

$$e_{01} = \tfrac{27}{125}\ ,\ e_{11} = 0\ ,$$
$$e_{02} = \tfrac{98}{125}\ ,\ e_{12} = -\tfrac{36}{125}\ ,\ \text{and}$$
$$a_{11} = -\tfrac{1}{4}\ ,\ a_{21} = 0\ ,\ a_{12} = \tfrac{1}{2}\ ,\ a_{22} = -\tfrac{1}{3}.$$



(v) Transposed direct form II

We take the direct form II derived in part (ii) and reverse the arrows as well as exchange the input and output. Then redrawing the flow graph, we get:



(b) **To get the difference equation for the flow graph of part (v) in (a), we first define the intermediate variables: $w_1[n]$, $w_2[n]$ and $w_3[n]$. We have:**

(b) To get the difference equation for the flow graph of part (v) in (a), we first define the intermediate variables: $w_1[n]$, $w_2[n]$ and $w_3[n]$. We have:

$$\begin{aligned}
(1)\ w_1[n] &= x[n] + w_2[n-1] \\
(2)\ w_2[n] &= \frac{1}{4}y[n] + w_3[n-1] - \frac{1}{5}x[n] \\
(3)\ w_3[n] &= -\frac{5}{24}y[n] - \frac{1}{12}y[n-1] \\
(4)\quad y[n] &= w_1[n].
\end{aligned}$$

Combining the above equations, we get:

$$y[n] - \frac{1}{4}y[n-1] + \frac{5}{24}y[n-2] + \frac{1}{12}y[n-3] = x[n] - \frac{1}{5}x[n-1].$$

Taking the Z-transform of this equation and combining terms, we get the following transfer function:

$$H(z) = \frac{1 - \frac{1}{5}z^{-1}}{1 - \frac{1}{4}z^{-1} + \frac{5}{24}z^{-2} + \frac{1}{12}z^{-3}}$$

which is equal to the initial transfer function.

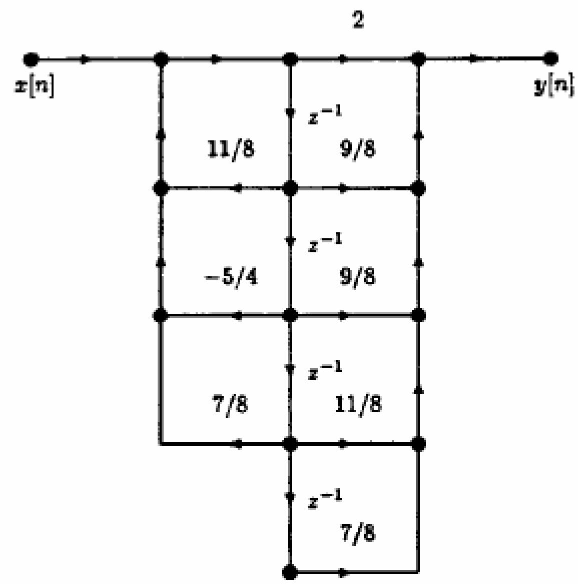
6.25. (a)

$$\begin{aligned} H(z) &= \frac{1}{1-z^{-1}}\left[\frac{1-\frac{1}{2}z^{-1}}{1-\frac{3}{8}z^{-1}+\frac{7}{8}z^{-2}}+1+2z^{-1}+z^{-2}\right] \\ &= \frac{2+\frac{9}{8}z^{-1}+\frac{9}{8}z^{-2}+\frac{11}{8}z^{-3}+\frac{7}{8}z^{-4}}{1-\frac{11}{8}z^{-1}+\frac{5}{4}z^{-2}-\frac{7}{8}z^{-3}}. \end{aligned}$$

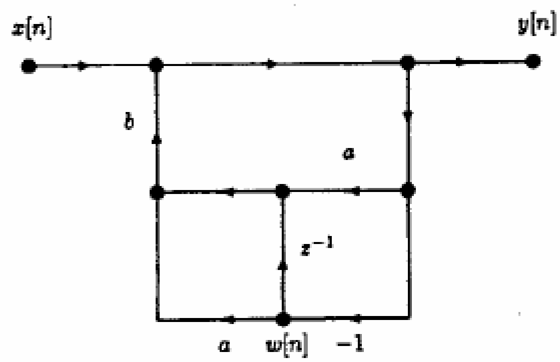
(b)

$$\begin{aligned} y[n] &= 2x[n]+\frac{9}{8}x[n-1]+\frac{9}{8}x[n-2]+\frac{11}{8}x[n-3]+\frac{7}{8}x[n-4] \\ &+ \frac{11}{8}y[n-1]-\frac{5}{4}y[n-2]+\frac{7}{8}y[n-3]. \end{aligned}$$

(c) Use Direct Form II:



6.28.



(a)

$$\begin{aligned} y[n] &= x[n] + abw[n] + bw[n-1] + aby[n] \\ w[n] &= -y[n]. \end{aligned}$$

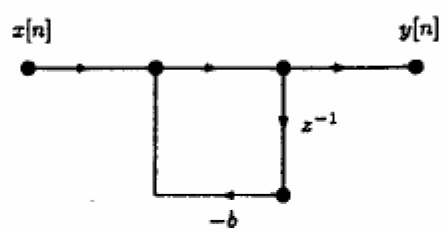
Eliminate w[n]:

$$\begin{aligned} y[n] &= x[n] - aby[n] - by[n-1] + aby[n] \\ y[n] &= x[n] - by[n-1] \end{aligned}$$

So:

$$H(z) = \frac{1}{1 + bz^{-1}}.$$

(b)



6.32. (a)

$$
\begin{aligned}
y_1[n] &= (1+r)x_1[n] + rx_2[n] \\
y_2[n] &= -rx_1[n] + (1-r)x_2[n].
\end{aligned}
$$

(b)

$$
\begin{aligned}
y_1[n] &= (1+a)x_1[n] + dx_2[n] \quad (a = r = d) \\
y_2[n] &= (1+cd)x_2[n] + abx_1[n] \quad (c = d = -1).
\end{aligned}
$$

(c)

$$
\begin{aligned}
y_1[n] &= (1+e)x_1[n] + ex_2[n] \quad (e = r) \\
y_2[n] &= efx_1[n] + (1+ef)x_2[n] \quad (f = -1).
\end{aligned}
$$

(d) B and C preferred over A:

(i) coefficient quantization. If r is small, $1+r$ may not be precisely representable even in floating point. Also, network A has 4 multipliers that must be quantized, while B and C have only 1.

(ii) computational complexity. Networks B and C require fewer multiplications per output sample.