1. Let

\[ y(n) = \sum_{k=-\infty}^{n} \sum_{l=-\infty}^{k} x(l) \]

Express \( Y(z) \) in terms of \( X(z) \). Simplify your results.

2. Let \( 0 < a < 1 \).

\[ H_1(z) = \frac{1-a}{1-az^{-1}}, \quad H_2(z) = \frac{1-a}{2} \frac{1+z^{-1}}{1-az^{-1}}. \]

What types of filters are they? Which one is better?

3. A stable digital filter has the following transfer function

\[ H(z) = \frac{z^4 + 1}{(z+.5)(z-.5)^2} \]

(a) Derive the poles/zeros of \( H(z) \) and sketch them on a pole/zero plot. State the ROC.

(b) Derive a stable \( h[n] \) from the given \( H(z) \).

4. Given

\[ y_1[n] = x[n] + 2y_1[n-1] - 2y_1[n-2] \]

\[ y_2[n] = y_1[n] - y_1[n-1] \]

(a) Derive \( H(z) = Y_2(z)/X(z) \)

(b) Derive a causal impulse response \( h[n] \).

(c) Provide a new set of difference equations such that a stable and causal system exists that has the same magnitude response as \( H(z) \).

5. Suppose a music signal \( s(t) \) has a cut-off frequency of 5 KHz. We obtain \( s_1(n) \) with a sampling freq. 10 KHz. After getting \( s_1(n) \), we find that the actual cut-off freq. for \( s(t) \) is in fact 3 KHz, therefore, we really only need to get \( s_2(n) \) with a sampling freq. 6 KHz. Explain how you can get \( s_2(n) \) from \( s_1(n) \) without using a D/C or C/D converter. Use a simple figure to show your approach.