1. Let

$$y(n) = \sum_{k=-\infty}^{n} \sum_{l=-\infty}^{k} x(l)$$

Express $Y(z)$ in terms of $X(z)$. Simplify your results.

2. Let $0 < a < 1$.

$$H_1(z) = \frac{1-a}{1-az^{-1}}, \quad H_2(z) = \frac{1-a}{2} \frac{1+z^{-1}}{1-az^{-1}}.$$

What types of filters are they? Which one is better?

3. A stable digital filter has the following transfer function

$$H(z) = \frac{z^4+1}{(z+.5)(z-.5)^2}$$

(a) Derive the poles/zeros of $H(z)$ and sketch them on a pole/zero plot. State the ROC.

(b) Derive a stable $h[n]$ from the given $H(z)$.

4. Given

$$y_1[n] = x[n] + 2y_1[n-1] - 2y_1[n-2]$$

$$y_2[n] = y_1[n] - y_1[n-1]$$

(a) Derive $H(z) = Y_2(z)/X(z)$

(b) Derive a causal impulse response $h[n]$.

(c) Provide a new set of difference equations such that a stable and causal system exists that has the same magnitude response as $H(z)$.

5. Suppose a music signal $s(t)$ has a cut-off frequency of 5 KHz. We obtain $s_1(n)$ with a sampling freq. 10 KHz. After getting $s_1(n)$, we find that the actual cut-off freq. for $s(t)$ is in fact 3 KHz, therefore, we really only need to get $s_2(n)$ with a sampling freq. 6 KHz. Explain how you can get $s_2(n)$ from $s_1(n)$ without using a D/C or C/D converter. Use a simple figure to show your approach.