(a) $x[n] = 5^n u[n]$:

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} h[k]x[n-k] \\ &= \sum_{k=-\infty}^{\infty} h[k]5^{(n-k)}u[n-k] \\ &= 5^n \sum_{k=-\infty}^{n} h[k]5^{-k} \end{aligned}$$

Becuase the summation depends on n, $x[n]$ is NOT AN EIGENFUNCTION.

(b) $x[n] = e^{j2\omega n}$:

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} h[k]e^{j2\omega(n-k)} \\ &= e^{j2\omega n} \sum_{k=-\infty}^{\infty} h[k]e^{-j2\omega k} \\ &= e^{j2\omega n} \cdot H(e^{j2\omega}) \end{aligned}$$

YES, EIGENFUNCTION.

(c) $e^{j\omega n} + e^{j2\omega n}$:

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} h[k]e^{j\omega(n-k)} + \sum_{k=-\infty}^{\infty} h[k]e^{j2\omega(n-k)} \\ &= e^{j\omega n} \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k} + e^{j2\omega n} \sum_{k=-\infty}^{\infty} h[k]e^{-j2\omega k} \\ &= e^{j\omega n} \cdot H(e^{j\omega}) + e^{j2\omega n} \cdot H(e^{j2\omega}) \end{aligned}$$

Since the input cannot be extracted from the above expression, the sum of complex exponentials is NOT AN EIGENFUNCTION. (Although, separately the inputs are eigenfunctions. In general, complex exponential signals are always eigenfunctions of LTI systems.)

(d) $x[n] = 5^n$:

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} h[k]5^{(n-k)} \\ &= 5^n \sum_{k=-\infty}^{\infty} h[k]5^{-k} \end{aligned}$$

YES, EIGENFUNCTION.

(e) $x[n] = 5^n e^{j2\omega n}$:

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} h[k]5^{(n-k)}e^{j2\omega(n-k)} \\ &= 5^n e^{j2\omega n} \sum_{k=-\infty}^{\infty} h[k]5^{-k}e^{-j2\omega k} \end{aligned}$$

YES, EIGENFUNCTION.

2.32. We first re-write the system function $H(e^{j\omega})$:

$$\begin{aligned} H(e^{j\omega}) &= e^{j\pi/4}\cdot e^{-j\omega}\left(\frac{1+e^{-j2\omega}+4e^{-j4\omega}}{1+\frac{1}{2}e^{-j2\omega}}\right) \\ &= e^{j\pi/4}G(e^{j\omega}) \end{aligned}$$

Let $y_1[n] = x[n] * g[n]$, then

$$\begin{aligned} x[n] &= \cos(\frac{\pi n}{2}) = \frac{e^{j\pi n/2}+e^{-j\pi n/2}}{2} \\ y_1[n] &= \frac{G(e^{j\pi/2})e^{j\pi n/2}+G(e^{-j\pi/2})e^{-j\pi n/2}}{2} \end{aligned}$$

Evaluating the frequency response at $\omega = \pm\pi/2$:

$$\begin{aligned} G(e^{j\frac{\pi}{2}}) &= e^{-j\frac{\pi}{2}}\left(\frac{1+e^{-j\pi}+4e^{-j2\pi}}{1+\frac{1}{2}e^{-j\pi}}\right) = 8e^{-j\pi/2} \\ G(e^{-j\frac{\pi}{2}}) &= 8e^{j\pi/2} \end{aligned}$$

Therefore,

$$y_1[n] = (8e^{j(\pi n/2-\pi/2)} + 8e^{j(-\pi n/2+\pi/2)})/2 = 8\cos(\frac{\pi}{2}n - \frac{\pi}{2})$$

and

$$y[n] = e^{j\pi/4}y_1[n] = 8e^{j\pi/4}\cos(\frac{\pi}{2}n - \frac{\pi}{2})$$

2.41. The input sequence,

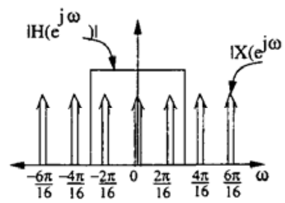
$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n + 16k],$$

has the Fourier representation

$$\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta[n + 16k] e^{-j\omega n} \\ &= \frac{1}{16} \sum_{k=-\infty}^{\infty} \delta(\omega + \frac{2\pi k}{16}). \end{aligned}$$

Therefore, the frequency representation of the input is also a periodic impulse train. There are frequency impulses in the range $-\pi \leq \omega \leq \pi$.

We sketch the magnitudes of $X(e^{j\omega})$ and $H(e^{j\omega})$:



From the sketch, we observe that the LTI system is a lowpass filter which removes all but three of the frequency impulses. To these, it multiplies a phase factor $e^{-j3\omega}$.

The Fourier transform of the output is

$$
\begin{aligned}
Y(e^{j\omega}) &= \frac{1}{16}\delta(\omega) + \frac{1}{16}e^{-j\frac{6\pi}{16}}\delta(\omega - \frac{2\pi}{16}) \\
&\quad + \frac{1}{16}e^{j\frac{6\pi}{16}}\delta(\omega + \frac{2\pi}{16})
\end{aligned}
$$

Thus the output sequence is

$$
y[n] = \frac{1}{16} + \frac{1}{8}\cos(\frac{2\pi n}{16} + \frac{3\pi}{8}).
$$

2.45. Let $x[n] = \delta[n]$, then

$$X(e^{j\omega}) = 1$$

The output of the ideal lowpass filter:

$$W(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = H(e^{j\omega})$$

The multiplier:

$$(-1)^n w[n] = e^{-j\pi n} w[n]$$

causes a shift in the frequency domain:

$$W(e^{j(\omega-\pi)}) = H(e^{j(\omega-\pi)})$$

The overall output:

$$y[n] = e^{-j\pi n} w[n] + w[n]$$
$$Y(e^{j\omega}) = H(e^{j(\omega-\pi)}) + H(e^{j\omega})$$

Noting that:

$$H(e^{j(\omega-\pi)}) = \begin{cases} 1, & \frac{\pi}{2} \le |\omega| \le \pi \\ 0, & |\omega| < \frac{\pi}{2} \end{cases}$$

$Y(e^{j\omega}) = 1$, thus $y[n] = \delta[n]$.

]

2.47. (a)

$$\begin{aligned} y[n] &= x[n] + 2x[n-1] + x[n-2] \\ &= x[n] * h[n] \\ &= x[n] * (\delta[n] + 2\delta[n-1] + \delta[n-2]) \\ h[n] &= \delta[n] + 2\delta[n-1] + \delta[n-2] \end{aligned}$$

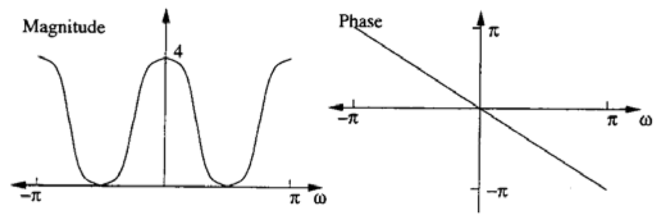
(b) Yes. $h[n]$ is finite-length and absolutely summable.

(c)

$$
\begin{aligned}
H(e^{j\omega}) &= 1 + 2e^{-j\omega} + e^{-2j\omega} \\
&= 2e^{-j\omega}(\frac{1}{2}e^{j\omega} + 1 + \frac{1}{2}e^{-j\omega}) \\
&= 2e^{-j\omega}(\cos(\omega) + 1)
\end{aligned}
$$

(d)

$$
\begin{aligned}
|H(e^{j\omega})| &= 2(\cos(\omega) + 1) \\
\angle H(e^{j\omega}) &= -\omega
\end{aligned}
$$



(e)

$$
\begin{aligned}
h_1[n] &= \frac{1}{2\pi}\int_{<2\pi>} H_1(e^{j\omega})e^{j\omega}d\omega \\
&= \frac{1}{2\pi}\int_{<2\pi>} H(e^{j(\omega+\pi)})e^{j\omega n}d\omega \\
&= \frac{1}{2\pi}\int_{<2\pi>} H(e^{j(\omega)})e^{j(\omega-\pi)n}d\omega \\
&= e^{-j\pi n}\frac{1}{2\pi}\int_{<2\pi>} H(e^{j(\omega)})e^{j\omega n}d\omega \\
&= -1^n h[n] \\
&= \delta[n] - 2\delta[n-1] + \delta[n-2]
\end{aligned}
$$

3.22. (a)

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} h[k]x[n-k] \\ &= \sum_{k=-\infty}^{\infty} \left(3\left(-\frac{1}{3}\right)^k u[k]\right) u[n-k] \\ &= \sum_{k=0}^{n} 3\left(-\frac{1}{3}\right)^k \\ &= \begin{cases} \frac{9}{4}\left(1-\left(-\frac{1}{3}\right)^{n+1}\right), & n \geq 0 \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

(b)

$$\begin{aligned} Y(z) &= H(z)X(z) \\ &= \frac{3}{1+\frac{1}{3}z^{-1}} \frac{1}{1-z^{-1}} \\ &= \frac{\frac{3}{4}}{1+\frac{1}{3}z^{-1}} + \frac{\frac{9}{4}}{1-z^{-1}} \\ y[n] &= \frac{3}{4}\left(-\frac{1}{3}\right)^n u[n] + \frac{9}{4}u[n] \\ &= \frac{9}{4}\left(1+\frac{1}{3}\left(-\frac{1}{3}\right)^n\right) u[n] \\ &= \frac{9}{4}\left(1-\left(-\frac{1}{3}\right)^{n+1}\right) u[n] \end{aligned}$$

3.23. (a)

$$\begin{aligned} H(z) &= \frac{1-\frac{1}{2}z^{-2}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} \\ &= -4 + \frac{5+\frac{7}{2}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} \\ &= -4 - \frac{2}{1-\frac{1}{2}z^{-1}} + \frac{7}{1-\frac{1}{4}z^{-1}} \\ h[n] &= -4\delta[n] - 2\left(\frac{1}{2}\right)^n u[n] + 7\left(\frac{1}{4}\right)^n u[n] \end{aligned}$$

3.27. (a)

$$
\begin{aligned}
X(z) &= \frac{1}{(1+\frac{1}{2}z^{-1})^2(1-2z^{-1})(1-3z^{-1})} \qquad \frac{1}{2} < |z| < 2 \\
&= \frac{\frac{1}{35}}{(1+\frac{1}{2}z^{-2})^2} + \frac{\frac{88}{1225}}{(1+\frac{1}{2}z^{-1})} - \frac{\frac{1568}{1225}}{(1-2z^{-1})} + \frac{\frac{2700}{1225}}{(1-3z^{-1})}
\end{aligned}
$$

Therefore,

$$x[n] = \frac{1}{35}(n+1)\left(\frac{-1}{2}\right)^{n+1}u[n+1] + \frac{58}{(35)^2}\left(\frac{-1}{2}\right)^{n}u[n] + \frac{1568}{(35)^2}(2)^n u[-n-1] - \frac{2700}{(35)^2}(3)^n u[-n-1]$$

(b)

$$X(z) = e^{z^{-1}} = 1 + z^{-1} + \frac{z^{-2}}{2!} + \frac{z^{-3}}{3!} + \frac{z^{-4}}{4!} + \ldots$$

Therefore, $x[n] = \frac{1}{n!}u[n]$.

(c)

$$X(z) = \frac{z^3 - 2z}{z-2} = z^2 + 2z + \frac{2}{1-2z^{-1}} \qquad |z| < 2$$

Therefore,

$$x[n] = \delta[n+2] + 2\delta[n+1] - 2(2)^n u[-n-1]$$

(

3.32. From the pole-zero diagram

$$X(z) = \frac{z}{(z^2 - z + \frac{1}{2})(z + \frac{3}{4})} \qquad |z| > \frac{3}{4}$$

$$\begin{aligned} y[n] &= x[-n+3] = x[-(n-3)] \\ &\Rightarrow\ Y(z) = z^{-3}X(z^{-1}) = \frac{z^{-3}z^{-1}}{(z^{-2} - z^{-1} + \frac{1}{2})(z^{-1} + \frac{3}{4})} \\ &= \frac{8/3}{z(2 - 2z + z^2)(\frac{4}{3} + z)} \end{aligned}$$

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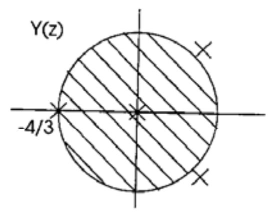
Poles at $0, -\frac{4}{3}, 1 \pm j$, zeros at ∞

$x[n]$ causal $\Rightarrow x[-n+3]$ is left-sided $\Rightarrow$ ROC is $0 < |z| < 4/3$.



3.40. (a) After writing the following equalities:

$$\begin{aligned} V(z) &= X(z) - W(z) \\ W(z) &= V(z)H(z) + E(z) \end{aligned}$$

we solve for W(z):

$$W(z) = \frac{H(z)}{1+H(z)}X(z) + \frac{1}{1+H(z)}E(z)$$

(b)

$$\begin{aligned} H_1(z) &= \frac{H(z)}{1+H(z)} = \frac{\frac{z^{-1}}{1-z^{-1}}}{1+\frac{z^{-1}}{1-z^{-1}}} = z^{-1} \\ H_2(z) &= \frac{1}{1+\frac{z^{-1}}{1-z^{-1}}} = 1 - z^{-1} \end{aligned}$$

(c) $H(z)$ is not stable due to its pole at $z = 1$, but $H_1(z)$ and $H_2(z)$ are.