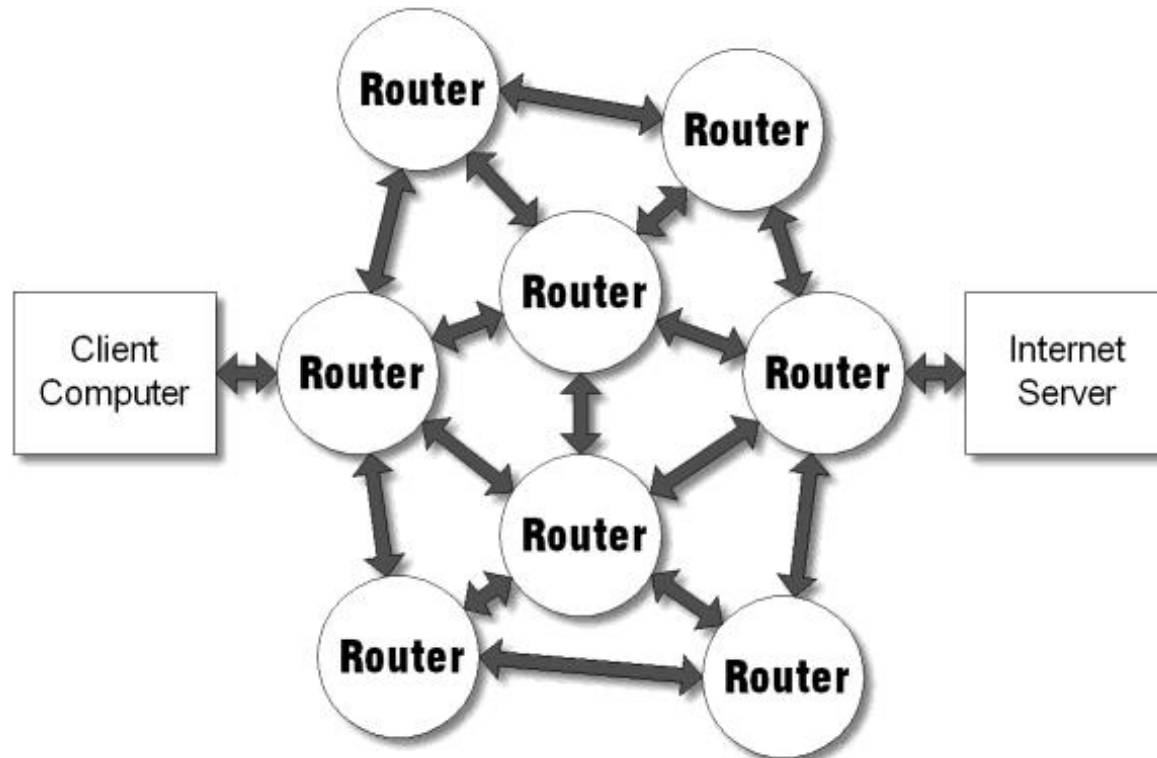


## Modeling of telecommunications network traffic

Traffic statistics are important for

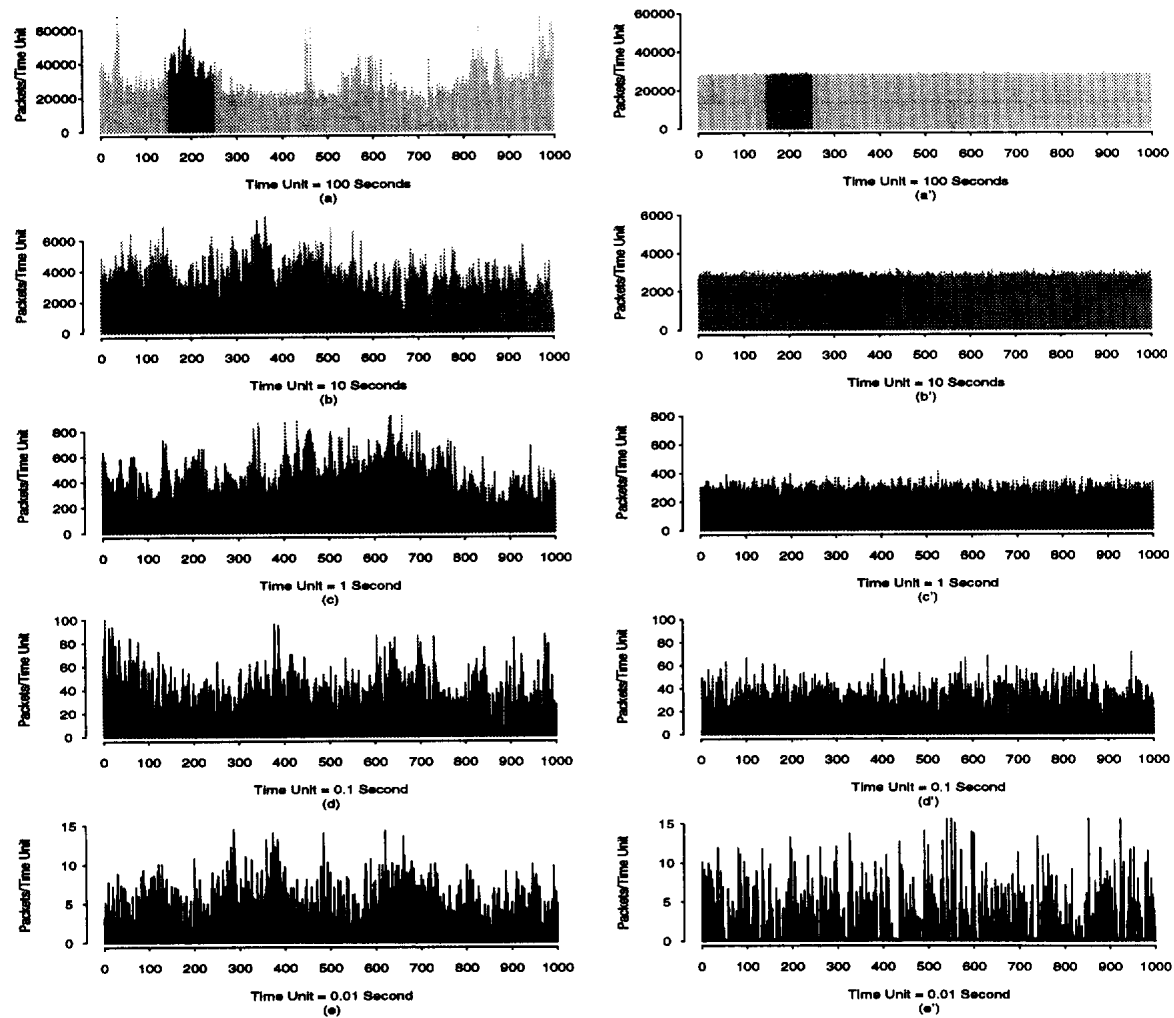
- evaluating performance of a network
- ensuring quality of service
- the design and sizing of electronic processing and switching systems



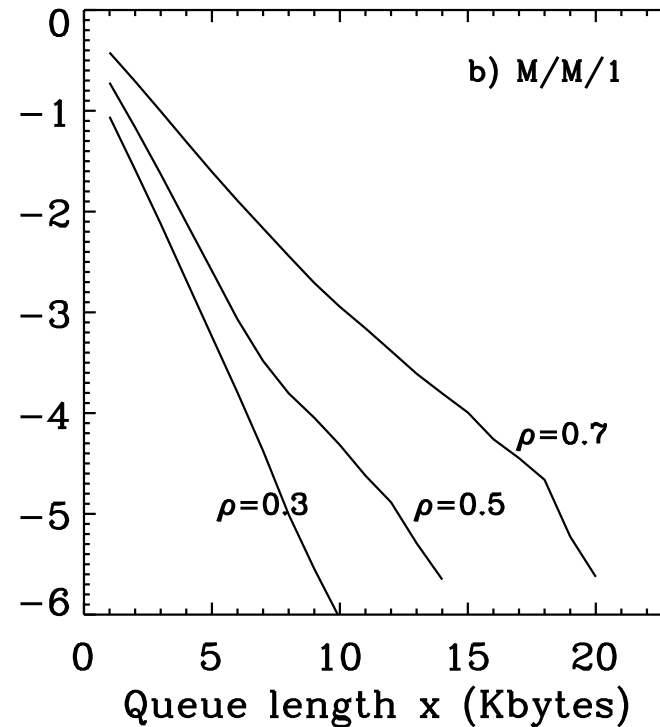
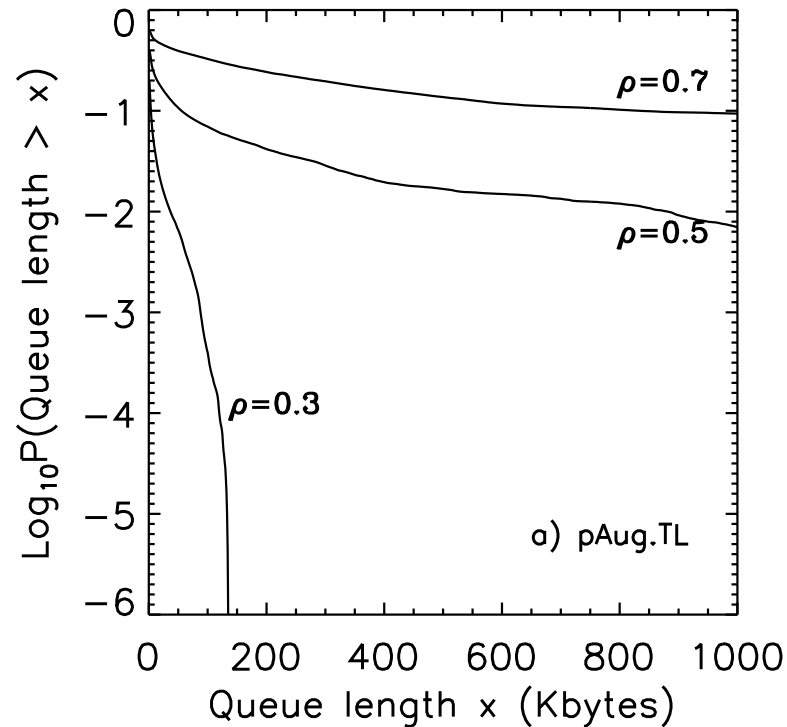
- Throughput
- Delay
- Delay jitter
- Packet loss prob.

# What is “Fractal Traffic”? varies over many or all time scales

Left: real traffic; right: Poisson model (Leland et al. (1994))



## Failure of Poisson modeling



When the buffer size is  $q_0$ , then  $\text{Prob}(\text{Queue length} > q_0)$  estimates the packet loss probability; Queue size/ $C$  + service time = delay

Poisson modeling underestimates buffer size, packet loss probability, or packet delay time by several orders of magnitude !!!

## Fractional Brownian motion (fBm) $B_H(t)$

- Gaussian process with mean 0 & stationary increments

- Variance:

$$E[(B_H(t))^2] = t^{2H}$$

- Power spectral density

$$f^{-(2H+1)}$$

- $H$ : Hurst parameter.

$1/2 < H < 1$ : long memory (long-range-dependence (LRD))

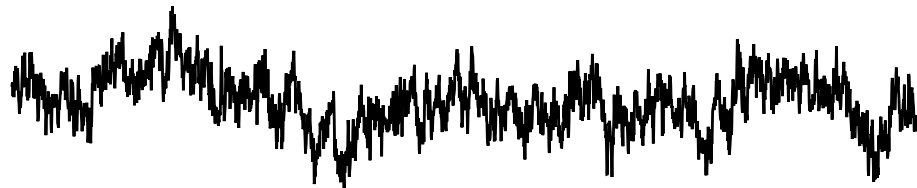
$H = 1/2$ : standard Brownian motion

$0 < H < 1/2$ : anti-persistence

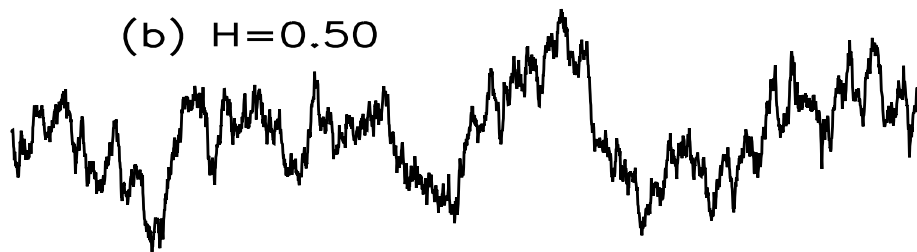
- Applications to a wide range of problems (including Hollywood movie making—fancy landscape)

## Examples of fBm processes with different $H$

(a)  $H=0.25$



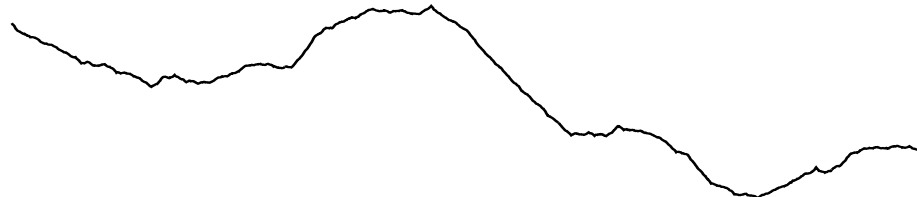
(b)  $H=0.50$



(c)  $H=0.75$



(d)  $H=0.90$



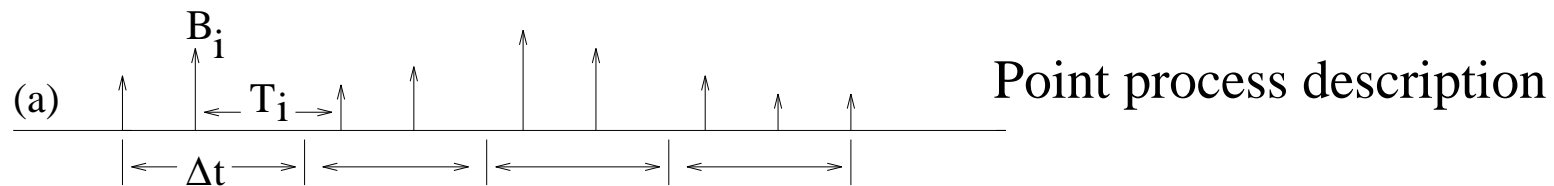
## Ubiquity of $1/f$ processes

- Many old references can be found in Press (1978), Bak (1996), Wornell (1996) (**Voss:  $1/f$ -music**)
- Increment process  $\{x_1, x_2, \dots, x_n\}$ : power spectral density  $f^{-(2H-1)}$ ;  
Random walk process  $\{y_n\}$ ,  $y_n = \sum_{i=1}^n x_i$ ,  $f^{-(2H+1)}$   
—  $H$  Hurst parameter
- Kolmogorov's turbulence energy spectrum  $E(f) \sim f^{-(2H+1)}$ ,  
 $H = 1/3$
- Engineering: noise in device; network traffic; power-outage
- Human cognition & coordination; DNA sequence; distribution of prime numbers; multistable visual perception; neuron inter-spike interval data (—, draw a line with the same length—error sequence is a  $1/f$  process)
- **Mechanism for  $1/f$  processes?**

## Modeling LRD traffic by the fBm process

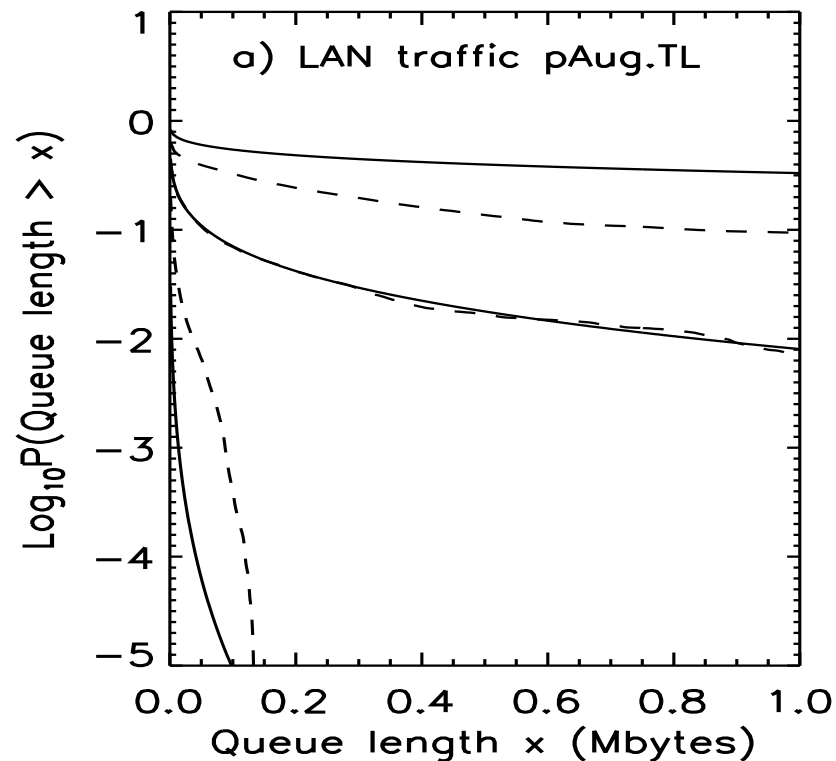
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- fBm is used to model counting process of network traffic
  - The mean traffic loading to a network in each time slot  $\Delta t$  is  $m$
  - The variation of traffic around  $m$  is modeled by fBm
- Two parameters are involved:  $H$  and a variance coefficient characterizing how significant this variation around the mean level is



# Performance of the fBm model as a long-range-dependent traffic model

(Gao & Rubin)



Better than Poisson model. But much room for further improvement!

(Monofractal models are not sufficient)