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# Financial crisis, Omori's law, and negative entropy flow

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ABSTRACT

The 2008 global financial crisis has revived great interest in early warning system (EWS) models for reducing the risks of future crises. Existing EWS models employ aggregated variables that cannot examine the nonlinear dynamics of participating players on scales smaller than a country in unstable, non-equilibrium economies. To help understand the mechanism of financial crises and identify new robust indicators for financial crises and economic recessions, in this work, we take an "anatomical" approach, i.e., to examine the income structures of different sectors of an economy separately, as well as to analyze the exposure networks associated with Fannie Mae/Freddie Mac, Lehman Brothers, and American International Group. We show that the losses in exposure networks can be modeled by a two-parameter Omori-law-like distribution for earthquake aftershocks. Such a distribution suggests that losses will be widespread around crises or recessions. Indeed, around crises or recessions, the heavy-tailed distributions for the negative income cluster are even heavier than those for the positive income cluster. Consequently, the entropies associated with the distribution of the negative income cluster exceed that of the positive income cluster. Moreover, instability propagates from the crisis initiating sector to other sectors. Therefore, the anatomical approach developed here can indeed shed some light on the detailed dynamics of financial crises and economic recessions, and the distribution and entropy approaches can help predict economic downturns.



## 1. Introduction

In the past three decades, the world has seen many countries experiencing financial crises with different degrees of severity. Especially costly is the 2008 global financial crisis, which has affected essentially all the industrialized countries, as well as a large number of developing economies. While the world is still recovering from the 2008 crisis, it is important to learn lessons from previous major financial crises (Aziz, Caramazza, & Salgado, 2000; BBC, 2007), develop quantitative theoretical models of crises (Dymski & Pollin, 1992; Jorge & Chen, 2002; Kindleberger, 1996; Minsky 1975, 1982), and create better early warning system (EWS) models for predicting and hopefully mitigating severity of future crises.

EWS models aim to anticipate whether and when individual countries may be affected by a financial crisis. There are various types of financial crises: currency crises, banking crises, sovereign debt crises, private sector debt crises, and equity market crises. Most EWS models focus primarily on currency crises (Bussiere & Fratzscher, 2002). Existing EWS models include the leading indicator approach (Kaminsky, Lizondo, & Reinhart, 1998), the discrete-dependent-variable approach based on logit and probit models (Bussiere & Fratzscher, 2002; Maddala, 1989), structural equation model (Goldberger, 1972), and network-based models (Niemiraa & Saaty, 2004; Ozkan-Gunay & Ozkan, 2007). The results obtained so far are mixed, however. For example, the loss of information using a binary variable in the leading indicator approach is questionable (Berg, Borensztein, & Patillo, 2004; Berg & Patillo, 1999, 2000). After carefully examining the Multiple-Indicator Multiple Cause (MIMIC) model of Goldberger (1972), Rose and Spiegel (2009) in particular, have concluded that few of the characteristics suggested as potential causes of the crisis actually help predict the intensity and severity of the crisis across countries. The best indicators for the 2008 crisis include asset price inflation, rising leverage, large sustained current account deficits, and a slowing trajectory of economic growth (Reinhart & Rogoff, 2008). Overall, however, economists have not had a particularly good track record at predicting the timing of crises (Rose & Spiegel, 2009).

In economics, an important assumption is the economic equilibrium — economic forces are balanced; in the absence of external influences, the equilibrium values of economic variables will not change (Solow, 1956; Swan, 1956; Turnovsky, 2000; Varian, 1992). While the validity of this assumption in normal healthy economic times may be debated (Ruelle, 1991), it clearly is violated during a crisis. Unfortunately, the variables employed by the afore mentioned EWS models are not quite viable for examining the nonlinear dynamics of participating players on scales smaller than a country in unstable, non-equilibrium economies, since they are aggregated variables.

To gain a deep understanding of the nonlinear dynamics of an unstable economy, especially the emerging large scale complex behavior, one has to look into the detailed interactions among the participating players of an unstable economy. This amounts to solidly bridging

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microeconomics with macroeconomics, and has to be considered a grand challenge in economics. However, time is not quite ripe to comprehensively examine the detailed interactions among the participating players of an economy. In this work, we take an intermediate approach, by focusing on the collective economic dynamics associated with losses or negative incomes. Specifically, we shall start from the exposure networks of Fannie Mae/Freddie Mac (FNM/FRE), Lehman Brothers (LEH), and American International Group (AIG). We then examine quarterly incomes of thousands of companies in 9 different sectors of the economy, hoping to find new robust indicators of crises or recessions. We will elucidate how instability propagates from the crisis initiating sector to other sectors. The proposed scheme, if works, will shed light not only on financial crises, but also on economic recessions in general.

To serve our goal, we have utilized two types of data. One is the amount of investments exposed to FNM, FRE, LEH, and AIG. Such data were obtained by exhaustively searching the relevant files on the Internet, and then extracting the amount of investments needed for our analysis. This is presented in Section 2. The second type of data is the quarterly incomes of thousands of companies, and thus are much bigger. We will describe the details in Section 3.

## 2. Distribution of losses in crisis exposure networks

The 2007–2009 financial crisis was a truly gigantic one, as it claimed Bear Stearns, FNM, FRE, LEH, AIG, and Washington Mutual (WaMu) in the United States, and spiraled to the entire world and consumed other businesses, including Japan's Yamato Life Insurance. It even has much to do with the current dire economic conditions in many countries in the European Union. Here, we report an effort of constructing networks exposed to AIG, LEH, and FNM/FRE, where the network nodes are the companies which invested in FNM, FRE, LEH, and/or AIG, and the strength of the links was characterized by the amount of the investment. Such data were obtained by thoroughly searching daily list of companies reporting exposures to AIG, LEH, and FNM/FRE, and then manually extracting the amount of investments from those files. This work took us about half year, from summer of 2008 to the end of 2008. We find that the distribution for the investments in these exposure networks is strikingly similar to the Omori's law for the rate of occurrence of aftershocks triggered by a main earthquake. The similarity stimulates us to take an evolutionary point of view and examine the entropy production associated with the formation of exposure networks.

The exposure networks we constructed contain 34, 151, and 146 companies worldwide, exposed to AIG, LEH, and FNM/FRE, respectively. Based on the foreign exchange rate data on October 8, 2008, the amount of investments ranges from less than 1 million to hundreds of millions or even tens of billions of US dollars in each network. AIG, LEH, and FNM/FRE, being connected to all the companies exposed to them, may be called hub-nodes, just as hubs of major airlines. In contrast, other companies may be called end-nodes, and the amount of their exposures to AIG, LEH, and FNM/FRE determines the strength of the links connecting them to the hub-nodes. To assess the severity of the collapse of hub-nodes, it is most important to find the number of companies with big investments on those hub-nodes. Mathematically, this amounts to estimating the complementary cumulative distribution function (CCDF),

$$P(X \geq x) = \text{Probability that } X \geq x \text{ million}.$$

The huge range of capitals involved in each network makes such a task feasible and meaningful.

To reliably estimate distributions from sparse data with wide range, we have used the technique of equal-log-bin (Gao, Cao, Tung, & Hu, 2007), by first taking logarithm of the data, then estimating the CCDF. The results are shown in Fig. 1. We observe that the CCDFs for AIG and LEH are concave to the origin, while that for FNM/FRE is a power-law.

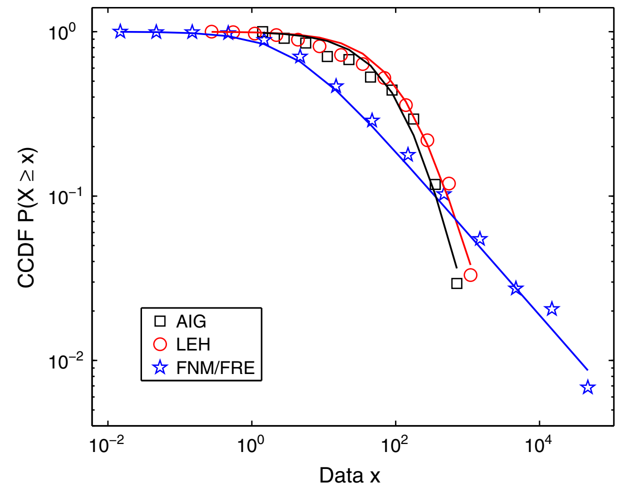


**Fig. 1.** CCDF for the investment distribution of the 3 exposure networks. (α, β) are (2, 118), (1.6, 116), and (0.5, 2), for AIG, LEH, and FNM/FRE, respectively.

The similarity between AIG and LEH suggests that they may have very similar business models/strategies. The power-law behavior for FNM/FRE suggests that the FNM/FRE exposure network is a type of power-law or scale-free network (Albert, Jeong, & Barabasi, 2000; Jeong, Mason, Barabasi, & Oltvai, 2001). The difference between FNM/FRE and AIG/LEH might partly be due to the fact that many companies had greatly reduced their investments on FNM/FRE before FNM/FRE were taken over by the U.S. government.

Interestingly, Scherer, Harhoff, and Kukies (2000) analyzed more than 10 datasets for profit outcomes from technological innovations and observed CCDFs very similar to those for AIG and LEH. This is not surprising though, since the formation of exposure networks is driven by pursuing greater profits by various parties.

Can we find a simple closed-form formula to fit all the CCDFs found here? The task sounds challenging, since the CCDFs for AIG and LEH are very different from that for FNM/FRE. Surprisingly, it turns out that an Omori-like law for earthquake aftershock activity solves the problem neatly. The Omori law for aftershock activity states that (Utsu, Ogata, & Matsu'ura, 1995),

$$r(t) = \frac{1}{\tau[1 + t/c]^p} \tag{1}$$

where $r(t)$ is the rate of occurrence of aftershocks with magnitudes greater than $m$ at time $t$ after a major earthquake, $\tau$ and $c$ are certain characteristic times, and $p > 0$ is the scaling parameter. The Omori law has been used to describe volatility return intervals in stock markets (Lillo & Mantegna, 2003; Weber, Wang, Vodenska-Chitkushev, Havlin, & Stanley, 2007). It has an interesting property that when $p \neq 1$, it essentially keeps its functional form with integration or differentiation. This motivates us to fit our CCDFs by the following two-parameter formula,

$$P(X \geq x) = \left(1 + \frac{x}{\beta}\right)^{-\alpha}, \tag{2}$$

where $\alpha > 0$ and $\beta > 0$ are parameters. The fitting is shown in Fig. 1 as red smooth curves, where $(\alpha, \beta)$ are (2, 118), (1.6, 116), and (0.5, 2), for AIG, LEH, and FNM/FRE, respectively. In all three cases, the fitting is remarkably good. This suggests that cascading of failures of financial institutions after the collapse of a major financial player is remarkably similar to the triggering of a sequence of aftershocks by a major earthquake.

Note that when $x \gg \beta$, Eq. (2) becomes a power-law,

$$P(X \geq x) \sim x^{-\alpha}. \tag{3}$$

When $\alpha < 2$, such a distribution is heavy-tailed having infinite variance. When $\alpha \leq 1$, the mean also becomes infinite. This is the case for FNM/FRE network. It is well known that a power-law network is very fragile when a hub-node is attacked. The large number of defaults in home mortgages and sharp decrease in home values are the very lethal attacks on FNM/FRE and many other financial institutions.

It is interesting to note that if we introduce a new random variable, $Y = X + \beta$, then $Y$ follows the Pareto distribution,

$$P(Y \geq y) = P(X \geq y - \beta) = \left(1 + \frac{y-\beta}{\beta}\right)^{-\alpha} = \left(\frac{\beta}{y}\right)^{\alpha}, \ y \geq \beta. \tag{4}$$

The ubiquity of Pareto and heavy-tailed distributions in economics (Gabaix, Gopikrishnan, Plerou, & Stanley, 2003; Rachev, Menn, & Fabozzi, 2005), and especially the similarity of our CCDFs (for AIG/LEH) to those for profit outcomes from technological innovations (Scherer et al., 2000), highly suggest that Eq. (2) is the right functional form for the investment distribution in exposure networks. The remaining fundamental question is to develop a proper model for the evolution of the exposure network and further justify Eq. (2).

To understand the evolution of exposure networks, let us consider a single company first. Suppose it engages in a number of businesses. Different business areas make different profits. The one with the highest profit will be privileged and expands rapidly. While attracting large investments, it also requires larger liquidity and costs to run it. In a profitable time, all parties will be happy, and investments will be enhanced. In a troubled time, however, the dominant business area may bring down the entire company. In fact, this is what had happened with WaMu and other financial institutions. For Wamu specifically, even though its deposit business was very good, losses from mortgages were too huge for it to survive in a harsh financial environment. This adaptive situation with positive feedbacks underlies the evolutionary character of an exposure network.

From a mathematical modeling perspective, we may assume that when a promising business area is just beginning, the situation is stable. The second law of thermodynamics states that entropy cannot decrease (Feynman, 1964). Therefore, the most stable situation is the one with the highest entropy. With respect to distributions, we can ask which distribution maximizes entropy. Here, since continuous distributions are concerned, the Shannon differential entropy is most relevant. It is defined as

$$H(f) = -\int f(x) \ln f(x) dx, \tag{5}$$

where $f(x)$ is the PDF. Given a target investment, we may assume that the mean investment $\bar{x}$ is given. Under this constraint, which investment distribution maximizes the Shannon entropy? It is the exponential distribution (Cover & Thomas, 1991). Its CCDF is

$$F(X \geq x) = e^{-\lambda x}, \quad x \geq 0 \tag{6}$$

and PDF is:

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0 \tag{7}$$

where $\lambda > 0$ is a parameter. Note that here, $\bar{x} = 1/\lambda$, and the Shannon entropy is

$$H_{\text{Exp}}(f) = 1 + \ln \bar{x}. \tag{8}$$

In contrast, when $\alpha > 1$, the entropy for the Omori-like distribution of Eq. (2) is

$$H_{\text{Omori}}(f) = 1 + \ln \bar{x} + 1/\alpha + \ln[(\alpha-1)/\alpha], \tag{9}$$

where we have assumed that the means for the two distributions are equal, and thus

$$\bar{x} = \beta/(\alpha-1). \tag{10}$$

Using Eq. (10) and Taylor series expansion, one can readily prove $1/\alpha + \ln[(\alpha - 1)/\alpha] < 0$.

To better understand the entropy maximizing property of the exponential distribution, it is beneficial to note that the distribution of molecules with the height in the atmosphere is an exponential distribution, and the fundamental Boltzmann law in statistical mechanics is also an exponential distribution (Feynman, 1964).

We now ask: How may evolution transform an exponential distribution to an Omori-like distribution of Eq. (2)? To find the answer, it is critical to notice that profits or losses will determine the level of investments in future. In other words, the mean investment $\bar{x} = 1/\lambda$ is a random variable — pursuing maximum profits is the whole purpose of this evolutionary process and will determine how $\bar{x}$ is adjusted from time to time.

Mathematically, treating $\lambda$ as a random variable with a PDF $f(\lambda)$ is equivalent to treating $\bar{x} = 1/\lambda$ as a random variable. Then the PDF for $x$ becomes

$$f(x) = \int_0^\infty \lambda e^{-\lambda x} f(\lambda) d\lambda \tag{11}$$

and CCDF is

$$F(X \geq x) = \int_x^\infty f(x) dx = \int_x^\infty \int_0^\infty \lambda e^{-\lambda x} f(\lambda) d\lambda. \tag{12}$$

Assuming uniform convergence, then the order of integration in Eq. (12) can be exchanged, and we obtain

$$F(X \geq x) = \int_0^\infty e^{-\lambda x} f(\lambda) d\lambda. \tag{13}$$

Therefore, $F(X \geq x)$ is the Laplace transform of $f(\lambda)$. When $F(X \geq x)$ is given by Eq. (2), $f(\lambda)$ is a Gamma distribution,

$$f(\lambda) = \frac{1}{\Gamma(\alpha)} \beta^{\alpha} \lambda^{\alpha-1} e^{-\beta\lambda}, \quad \lambda \geq 0, \quad \alpha > 0, \quad \beta > 0, \tag{14}$$

where $\Gamma(\alpha)$ is the Gamma function, and $\alpha$ and $\beta$ are parameters.

A special case of Gamma distribution is the chi-square distribution of degree $n$,

$$p(\lambda) = \frac{1}{2^{n/2}\Gamma(n/2)} \lambda^{n/2-1} e^{-\lambda/2} I_{\{\lambda \geq 0\}}, \tag{15}$$

where $\Gamma(n/2)$ is the Gamma function, and $I_{\{\lambda \geq 0\}}$ is 1 when $\lambda \geq 0$ and 0 otherwise. Note that chi-square distribution is the distribution for the summation of $n$ independent, standard normal random variables,

$$Q = \sum_{i=1}^{n} X_i^2.$$

Except for a constant scaling coefficient, $Q$ amounts to the total energy of a mechanical system, such as an assembly of non-interacting identical polymers with $n$ degrees of freedom considered in Chakraborti and Patriarca (2009). Among the distributions with given variance (or energy), Gaussian distribution maximizes the Shannon (and equivalently, the Boltzmann) entropy (Gao et al., 2007). Therefore, chi-square distribution is the distribution that maximizes the entropy of the compound system, a property called the variational principle in Chakraborti and Patriarca (2009). We may conclude that the more general Gamma distribution is the distribution that maximizes the entropy of the compound system with fractional number of degrees of freedom.

Since here entropy is associated with the total energy of a compound system, we need to ask what the analogy to energy is in economics. While this is impossible to precisely pinpoint, it is interesting to note that in the emerging new field, econophysics, which tries to develop a thermodynamic analogy for economy, energy is associated with capital (Mimkes, 2010).

## 3. Distribution of losses around recession times

The Omori-like law described by Eq. (2), while providing an excellent model for the loss distribution in exposure networks, also strongly suggests that losses around recession times will be widespread. To find the extent of losses around recession times, important clues may be gained by systematically examining quarterly incomes of thousands of companies in different sectors of the economy.

To obtain the relevant data, we first prepared a large list of stock symbols for thousands of U.S. companies in 9 sectors: Financial, Consumer Goods, Consumer Services, Basic Materials, Health Care, Industrials, Oil/Gas, Tech-Telecommunications, and Utilities. Part of the stock symbols was directly obtained from COMPUSTAT data base. It turns out that the symbol list there was smaller than that provided by Google Finance (http://finance.google.com/). We thus enlarged the stock symbol list by Google Finance. We then used these stock symbol lists as input to COMPUSTAT, and obtained quarterly incomes of thousands of companies. Since prior to 1990, income data for many companies in these 9 sectors were incomplete for meaningful analysis, they were not analyzed here.

In this study, we focus on pretax quarterly incomes. Although a pretax quarterly income of a company may be contributed by many business areas of a company, with some making money and some losing money, here, when a pretax quarterly income is positive, we call it a positive income. When a pretax quarterly income is negative, we call it a negative income. This ought not to be confused with contributions from individual components of a company. In fact, COMPUSTAT does not provide data about contributions from individual components of a company. In the analyses below, we consider negative and positive incomes as separate clusters. When the number of companies in each category or cluster is sufficiently large, distributional analysis is feasible.

A major result on the distribution of positive and negative incomes, which was reported in Gao et al. (2011), is reproduced in Fig. 2, where the black circles are illustrative of positive income distributions in general, and the straight lines in the tail region of the log–log

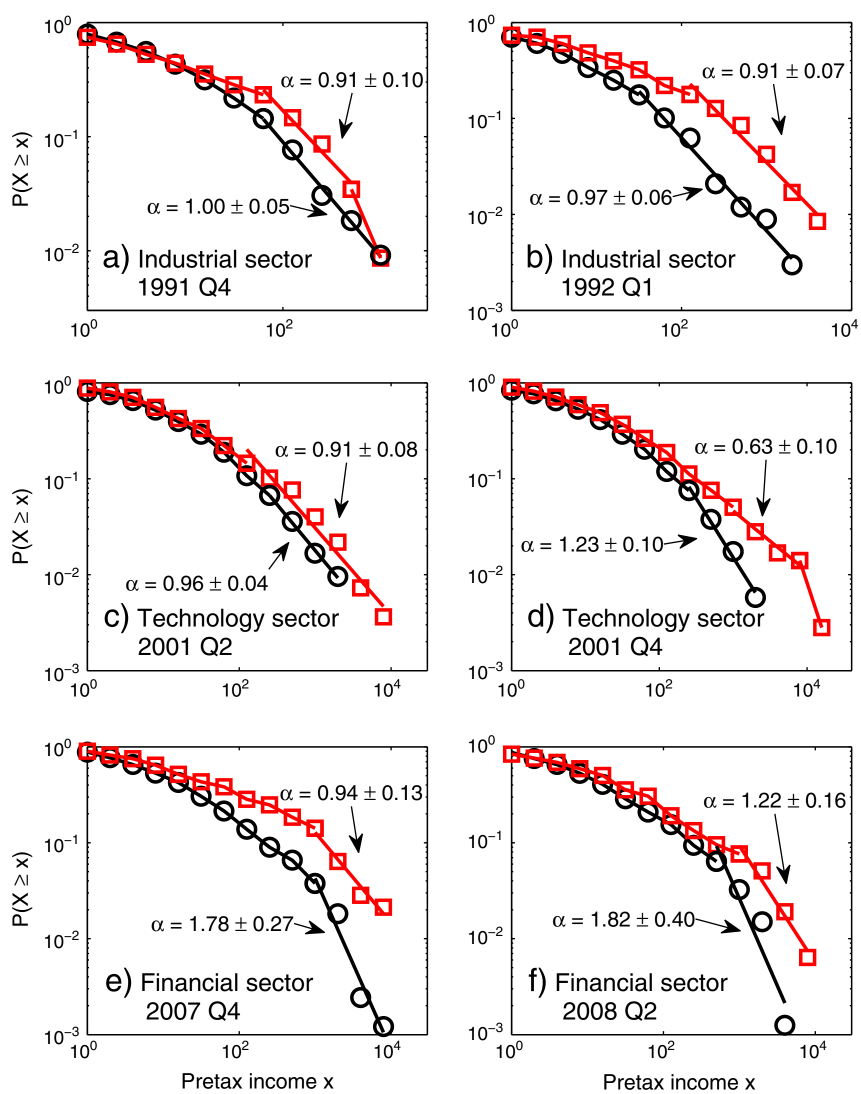


**Fig. 2.** CCDF (log–log scale) for negative (red square) and positive (black circle) pretax incomes among U.S. companies: (a) industrial sector, fourth quarter of 1991, 116 and 327 negative and positive pretax incomes; (b) industrial sector, first quarter of 1992, 118 and 335 negative and positive pretax incomes; (c) technology sector, second quarter of 2001, 275 and 418 negative and positive pretax incomes; (d) technology sector, fourth quarter of 2001, 356 and 345 negative and positive pretax incomes; (e) financial sector, fourth quarter of 2007, 141 and 823 negative and positive pretax incomes; (f) financial sector, second quarter of 2008, 157 and 800 negative and positive pretax incomes. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

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plots clearly indicate a heavy-tailed distribution. While the shape of the positive income distribution is largely independent of the economic health of the sector, the parameter α is not universal; instead, it correlates well with the health of the economy — it attains a larger value when a recession is more serious.

The shape of distribution of negative incomes, in contrast, strongly depends on the economic health of the sector. During healthy periods, negative incomes occur because of nonsystemic reasons (e.g., poor management) and are rare and small. When losses are very few, the loss distribution may not be well-defined; when losses become slightly more numerous, $X = -Y$ (where $Y < 0$ denotes negative incomes) may roughly follow an exponential distribution described by Eq. (6), or, with even more numerous losses, resemble a heavy-tailed distribution described by Eq. (3).

For the 2008 Financial Crisis, prior to the third quarter of 2007, the distribution of losses in any quarter is thinner than that of positive incomes in the same quarter (i.e., the tail of the loss distribution lies beneath the tail of the profit distribution). However, near the onset of and during a crisis, the distributions of negative incomes not only have also become Pareto, but have even heavier tails than those of positive incomes during the same period (Fig. 2(e,f)). Again, such behaviors are also seen in other recessions (Fig. 2(a–d)). Therefore, this distribution-based indicator can not only forewarn financial crises, but also monitor economic recessions in general.

Two important conclusions can be deduced from Fig. 2: (i) the shape of the distribution for the losses indicates the severity of the losses; and (ii) a recession may be defined as the instance that the distribution for the negative incomes is on top of that for the positive incomes. With these understandings, we can check when the recent U.S. recession ended, and more importantly, the health of the current U.S. economy. The basic results are shown in Figs. 3–5, where we observe that the recession ended around Q2 of 2009. Unfortunately, the losses, up to Q2 of 2011, have still been significant.

## 4. Entropy flow associated with losses

In deriving the Omori-like distribution of Eq. (2), we started from an exponential distribution. We now examine entropy change associated with the transition from an exponential to an Omori-like distribution. The answer can be obtained by comparing Eqs. (8) and (9). For ease of presentation, let us now consider discretized time. Denote the mean investment at $t = 0$ by $\overline{x_0}$. At $t = i$, after weighing profits and losses, the mean investment becomes $\overline{x_i}$. Let us denote the ratio between $\overline{x_0}$ and $\overline{x_i}$ by $r_i$, $\overline{x_i} = r_i\overline{x_0}$. Therefore, the profit is given by $r_i - 1$. Using Eqs. (8) and (9), we then find that the entropy change is

$$\Delta H_i = \ln r_i + 1/\alpha + \ln[(\alpha - 1)/\alpha]. \tag{16}$$

The term $\ln r_i$ is directly related to profits or losses. The terms $1/\alpha + \ln[(\alpha - 1)/\alpha$ are due to distributional changes of the investments and may be termed entropy change due to structural changes in a business. Since $1/\alpha + \ln[(\alpha - 1)/\alpha < 0$, to make total entropy change non-negative, $\ln r_i$ has to be large enough. Take AIG and LEH for examples, where $\alpha = 2$ and 1.6, respectively. Then $1/\alpha + \ln[(\alpha - 1)/\alpha = -0.19315$ and $-0.35583$. When $\Delta H_i = 0$, $r_i = 1.2131$ and 1.4274, respectively. While such profits may be realizable in good business times, in bad times with losses, $\ln r_i$ itself can be negative, making entropy change more negative and greatly aggravating instability.

As with income distribution analysis, it is more convenient to consider the entropy of the positive and negative income clusters separately. The stability or strength of an income cluster may be

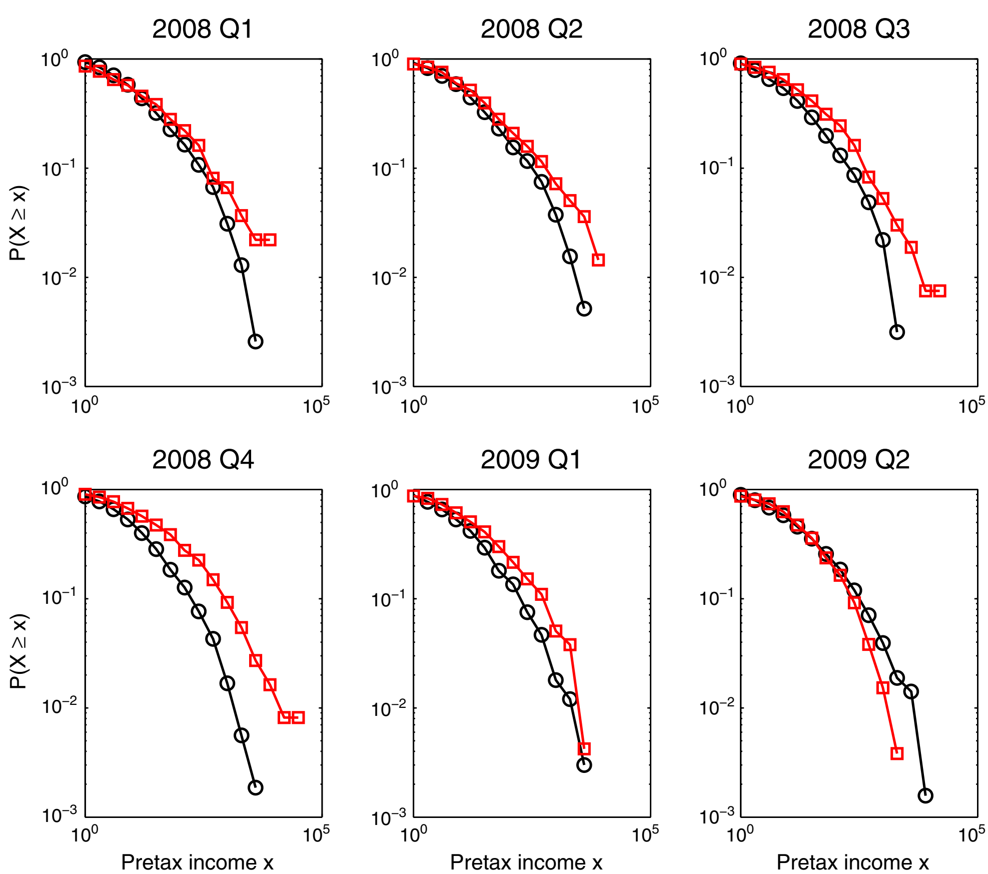


**Fig. 3.** CCDF (log–log scale) for negative (red square) and positive (black circle) pretax incomes among U.S. companies in Financial sector from 2008 Q1 to 2009 Q2. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

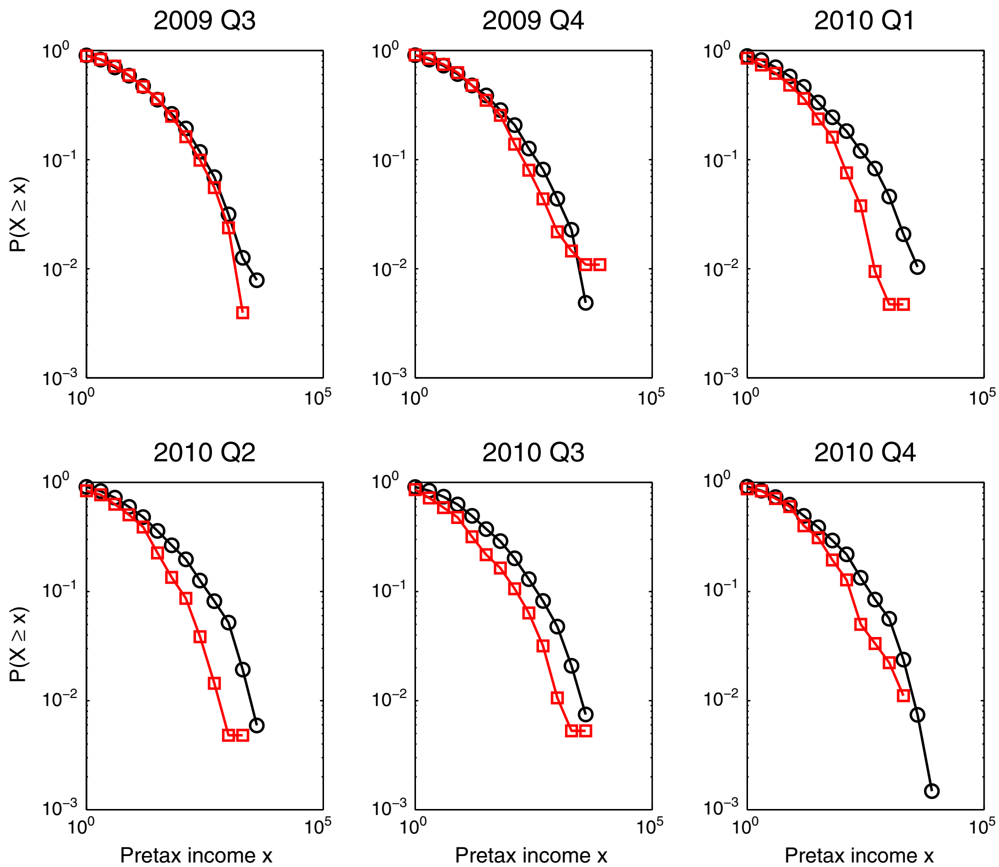


**Fig. 4.** CCDF (log–log scale) for negative (red square) and positive (black circle) pretax incomes among U.S. companies in Financial sector from 2009 Q3 to 2010 Q4. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

quantified by its entropy, given that the second law of thermodynamics can be equivalently restated as saying that the most stable configuration is the one with the highest entropy (Feynman, 1964). While Eq. (5) is most convenient for theoretical analysis, for discrete probabilities estimated from data, the following formula is more convenient (Cover & Thomas, 1991):

$$H = -\sum P_i \log P_i, \tag{17}$$

where $P_i$ are the probabilities that the positive or negative incomes will fall within a prescribed bin $i$ (where a bin is an interval of fixed length). We shall take 2 as the base of the logarithm so that the unit of the entropy is the bit. Note that when all the probabilities are equal, Shannon entropy attains its largest value; such a situation may be associated with the discretization of a uniform distribution. For illustrative purpose, we have reproduced a result from Gao et al. (2011) as Fig. 6, which shows $H$ from the first quarter of 2006 to the fourth quarter of 2008, for positive (black circles) and negative (red squares) incomes in 5 sectors (Financial, Consumer Goods, Consumer Services, Technology, and Health Care). The discrete probabilities in Fig. 6 are computed using a bin size of \$15 million. Tests using bin sizes of \$5, \$10, and \$20 million shifted the curves vertically, but the difference between $H$ for positive and negative incomes is largely independent of the bin size. The entropy of the distribution of positive incomes is almost constant, for all five

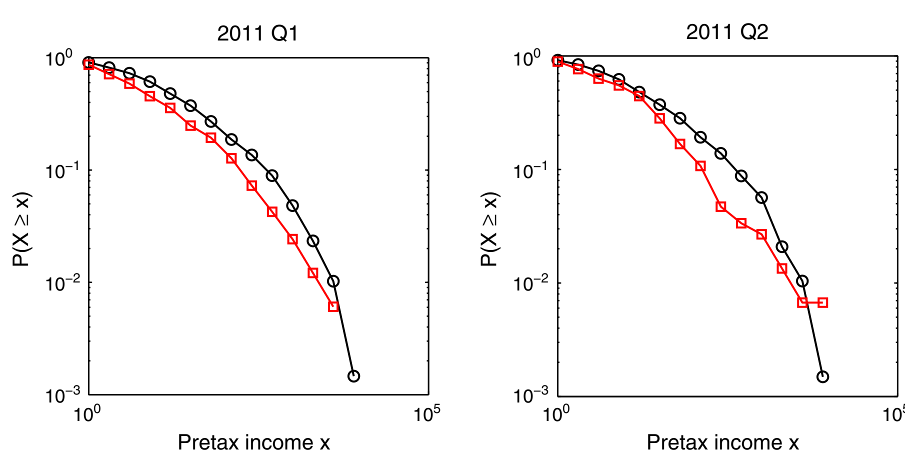


**Fig. 5.** CCDF (log–log scale) for negative (red square) and positive (black circle) pretax incomes among U.S. companies in Financial sector in 2011 Q1 and Q2. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

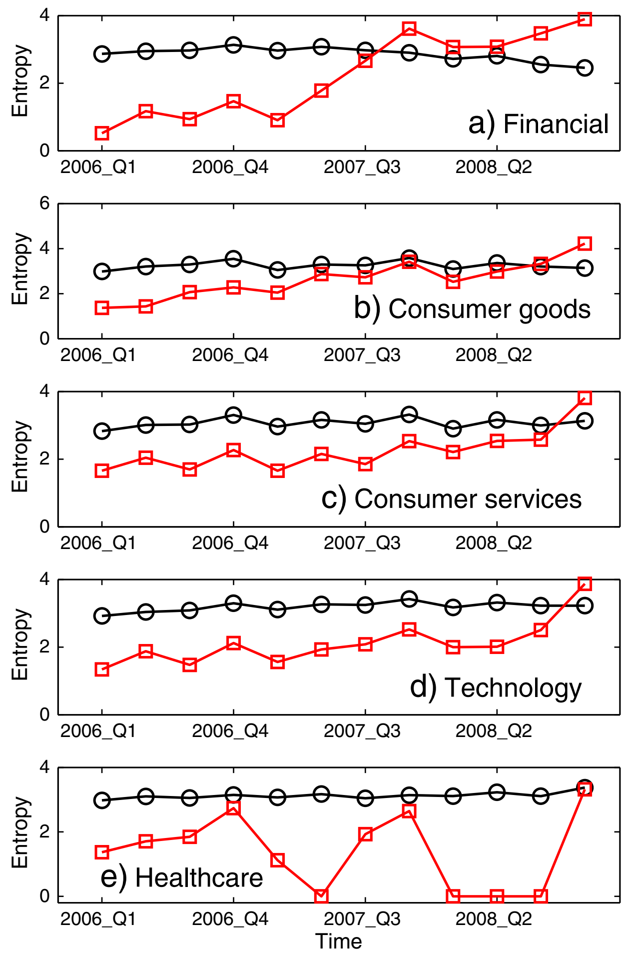


**Fig. 6.** Entropies (in units of bits) for the distribution of positive (black circles) and negative (red squares) incomes from the first quarter of 2006 to the fourth quarter of 2008 for 5 sectors. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

sectors examined here. In contrast, the entropy of the distribution of negative incomes varies considerably with time. For example, for the Financial sector, it is markedly smaller than the entropy of positive incomes until the third quarter of 2007, when the entropy rises sharply. By the third quarter of 2007, the difference between the two entropies is almost zero, suggesting that the cluster of financial companies with losses is almost as strong (i.e., this configuration is nearly as stable) as the cluster of profitable financial companies, and signals weakness in the sector. From the third quarter of 2007 on, the entropy of the distribution of negative incomes is noticeably larger than for positive incomes, indicating that the cluster of financial companies with negative incomes is well-established and stronger than the cluster of financial companies with positive incomes, consistent with the progression of the 2008 Financial Crisis.

The time series of entropy $H$ for other sectors in the period preceding and during the 2008 Financial Crisis show behavior consistent with the leading role of the Financial sector in the crisis, and propagation of the crisis from that sector into other sectors of the economy. Consumer Services (Fig. 6(c)) and Technology (Fig. 6(d)), for instance, do not show noticeable weakness until after third quarter of 2008. The temporal variations of entropy for Consumer Goods (Fig. 6(b)) show similar behavior, though in the second quarter of 2007 the negative income entropy nearly surpasses positive income entropy, suggesting the beginning of sector weakness at that time.

Note that the propagation of weakness from one sector to another during the 2008 Financial Crisis, as revealed in positive and negative income entropy, is also seen during other periods of market decline, such as the 1999–2003 technology stock fueled decline and the early 1990 recession (Gao et al., 2011). These results compare well with the National Bureau of Economic Research (NBER)'s business cycle contraction onset dating determinations during this period. The NBER business cycle determinations are, however, retrospectives. If we compare the dates of our entropy-based downturn onset identifications with the dates the NBER *announced* their onset identifications, our entropy-based identifications precede the NBER announcements; for the 2008 crisis, the entropy-based identification occurs 1 year earlier, and is fully consistent with the best results reported in the literature (Reinhart & Rogoff, 2008).

It is interesting to check how healthy the current U.S. financial sector is, using the entropy concept. The result is shown in Fig. 7. We see that the recession ended on Q2 of 2009; however, fragility still exists in the financial sector, since entropy for the negative incomes, though smaller than that of positive incomes, is still far greater than 0.

## 5. Discussions

The 2008 global financial crisis has led to renewed interests in EWS models for reducing the risks of future crises. So far, however, economists have not had a particularly good track record at predicting the timing of crises (Rose & Spiegel, 2009). One important reason may be that the existing EWS models employ aggregated variables that are not quite capable of examining the nonlinear dynamics of participating players on scales smaller than a country in unstable, non-equilibrium economies. While it is most desirable to deduce all the emerging large scale complex behaviors in crises economies from the detailed interactions among the participating players, the time might not be ripe for such a grand challenge. We thus have taken an intermediate approach to mitigate the problem with existing aggregated variables based EWS models. To achieve our goal, we have focused on the collective economic dynamics associated with losses or negative incomes. Specifically, we have examined the exposure networks of Fannie Mae/Freddie Mac, Lehman Brothers, and American International Group. We have found that the losses associated with them can be modeled by an Omori-law-like distribution for earthquake aftershocks. By considering the evolution of the exposure network and employing an optimization procedure, we have shown that the Omori-law-like distribution can be analytically derived. More importantly, the Omori-law-like exposure distributions suggest that around crises times, losses in an economy will

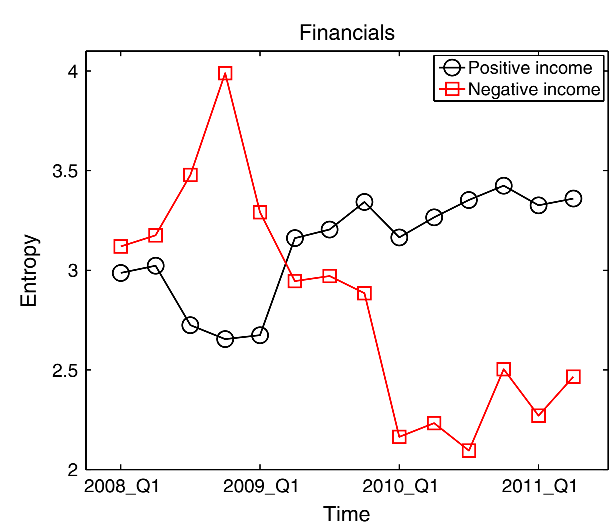


**Fig. 7.** Entropies (in units of bits) for the distribution of positive (black circles) and negative (red squares) incomes from the first quarter of 2008 to the second quarter of 2011 for Financial sector. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

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be widespread. By clustering pretax incomes of thousands of firms in 9 sectors of the U.S. economy into two categories, positive and negative, we have found that the distribution for the negative income cluster may develop into heavy-tailed distributions when economic instability enhances. In fact, a very reliable indicator for the onset of a crisis is when the distribution for the negative income cluster becomes heavier than that for the positive income cluster. Equivalently, this may be expressed as entropy associated with the negative income cluster which exceeds that of the positive income cluster.

Note that different sectors of an economy may be considered as an anatomy of the economy. Examining different sectors of an economy separately thus can indeed reveal certain features of the emerging large scale complex behaviors of an unstable economy. Indeed, such an approach can aptly reveal how instability propagates from crisis initiating sector to other sectors, which somewhat mimics the spread of a cancer from one part of a body to other parts. The accuracy and robustness of our distribution and entropy based indicators for predicting financial crisis suggests that such a more detailed approach is indeed promising. In fact, this approach may also be used for predicting general economic recessions, not just financial crises.

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## References

Albert, R., Jeong, H., & Barabasi, A. -L. (2000). Error and attack tolerance of complex networks. *Nature*, *406*, 378–482.

Aziz, J., Caramazza, F., & Salgado, R. (2000). Currency crises. *Search of common elements. IMF working paper*, *67*, Washington, DC: International Monetary Fund.

BBC (2007). Financial crises: Lessons from history. http://news.bbc.co.uk/2/hi/business/6958091.stm

Berg, A., & Patillo, C. (1999). Are currency crises predictable? *A test. IMF Staff Papers*, *Vol. 46* No. 2.

Berg, A., & Pattillo, C. (2000). *The challenges of predicting economic crises.* IMF Economic Issues, No. 22.

Berg, A., Borensztein, E., & Pattillo, C. (2004). *Assessing early warning systems: How have they worked in practice?* IMF Working Paper no. WP/04/52.

Bussiere, M., & Fratzscher, M. (2002). *Towards a new early warning system of financial crises.* European Central Bank Working Paper No.145.

Chakraborti, A., & Patriarca, M. (2009). Variational principle for the Pareto power law. *Physical Review Letters*, *103*, 228701.

Cover, T. M., & Thomas, J. A. (1991). *Elements of information theory.* New York, NY: Wiley.

Dymski, G., & Pollin, R. (1992). Hyman Minsky as hedgehog: The power of the Wall Street paradigm. In S. Fazzari, & D. B. Papadimitriou (Eds.), *Financial conditions and macroeconomic performance: Essays in honor of Hyman P. Minsky* (pp. 27–62) Armonk, New York: M.E. Sharpe.

Feynman, R. P. (1964). *The Feynman Lectures on Physics*, *Vol. 1*, : Addison-Wesley.

Gabaix, X., Gopikrishnan, P., Plerou, V., & Stanley, H. E. (2003). A theory of power-law distributions in financial market fluctuations. *Nature*, *423*, 267–270.

Gao, J. B., Cao, Y. H., Tung, W. W., & Hu, J. (2007). *Multiscale analysis of complex time series — Integration of chaos and random fractal theory, and beyond.* Hoboken, NJ: Wiley, 38–40 (51).

Gao, J. B., Hu, J., Mao, X., Zhou, M., Gurbaxani, B., & Lin, J. W. -B. (2011). Entropies of negative incomes, Pareto-distributed loss, and financial crises. *PLoS ONE*, *6*(10), e25053. http://dx.doi.org/10.1371/journal.pone.0025053.

Goldberger, A. S. (1972). Structural equation methods in the social sciences. *Econometrica*, *40*, 979–1001.

Jorge, A. C. -L., & Chen, Z. H. (2002). A theoretical model of financial crisis. *Review of International Economics*, *10*, 53.

Jeong, H., Mason, S. P., Barabasi, A. -L., & Oltvai, Z. N. (2001). Lethality and centrality in protein networks. *Nature*, *411*, 41–42.

Kaminsky, G. L., Lizondo, S., & Reinhart, C. M. (). Leading indicators of currency crises. *IMF Staff Papers*, *45, No. 1*. (pp. 1–48).

Kindleberger, C. (1996). *Manias, panics, and crashes: A history of financial crises* (3rd ed.)New York: Wiley.

Lillo, F., & Mantegna, R. N. (2003). Power-law relaxation in a complex system: Omori law after a financial market crash. *Physical Review E*, *68*, 016119.

Maddala, G. S. (1989). *Limited-dependent and qualitative variables in Econometrics.* Cambridge University Press: Econometric Society Monographs.

Mimkes, J. (2010). Stokes integral of economic growth: Calculus and Solow model. *Physica A*, *389*, 1665–1676.

Minsky, H. P. (1975). *John Maynard Keynes.* New York: Columbia University Press.

Minsky, H. P. (1982). *Can "It"' happen again?* New York: M.E. Sharpe.

Niemiraa, M. P., & Saaty, T. L. (2004). An analytic network process model for financial-crisis forecasting. *International Journal of Forecasting*, *20*, 573.

Ozkan-Gunay, E. N., & Ozkan, M. (2007). Prediction of bank failures in emerging financial markets: An ANN approach. *The Journal of Risk Finance*, *8*, 465.

Rachev, S. T., Menn, C., & Fabozzi, F. J. (2005). *Fat-tailed and skewed asset return distributions: Implications for risk management, portfolio selection, and option pricing.* Hoboken, NJ: Wiley.

Reinhart, C. M., & Rogoff, K. S. (2008). Is the 2007 US sub-prime financial crisis so different? An international historical comparison. *American Economic Review*, *98*, 339–344.

Rose, A. K., & Spiegel, M. M. (2009). *Cross-country causes and consequences of the 2008 crisis: Early Warning.* CEPR Discussion Paper 7354.

Ruelle, D. (1991). Chance and chaos. Princeton University Press: Princeton pp. 84-85.

Scherer, F. M., Harhoff, D., & Kukies, J. (2000). Uncertainty and size distribution of rewards from innovation. *Evolutionary Economics*, *10*, 175–200.

Solow, R. (1956). A contribution to the theory of economic growth. *Quarterly Journal of Economics*, *70*, 65–94 (The MIT Press).

Swan, T. (1956). Economic growth and capital accumulation. *Economic Record*, *32*(63), 334–361.

Turnovsky, S. J. (2000). *Methods of macroeconomic dynamics.* MIT Press.

Utsu, T., Ogata, Y., & Matsu'ura, R. S. (1995). The centenary of the Omori formula for a decay law of aftershock activity. *Journal of Physics of the Earth*, *43*, 1–33.

Weber, P., Wang, F., Vodenska-Chitkushev, I., Havlin, S., & Stanley, H. E. (2007). Relation between volatility correlations in financial markets and Omori processes occurring on all scales. *Physical Review E*, *76*, 016109.

Varian, H. R. (1992). *Microeconomic analysis* (3rd ed.) New York: Norton.

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