Multifractal analysis and modelling of VBR video traffic

Jianbo Gao and I. Rubin

A multiplicative multifractal model for variable bit rate video traffic is presented. The model simultaneously captures short and long range dependence into account. It only contains one parameter, which can be readily estimated through multifractal analysis of the trace data. It is shown that the model can accurately capture the behavior of a queueing system driven by actual video traffic over a wide range of utilization levels.

Introduction: Owing to the increasing demand on video services, variable bit rate (VBR) video traffic modelling has become a critical issue. Simple and accurate video traffic models are needed to solve problems arising from multimedia traffic management, such as traffic policing, shaping, and call admission control. Many services (such as video on demand and broadcast TV) impose bursty and bursty modulated with the measured traffic data. This is the smallest time scale resolvable by the measured trace data.

Given the weight sequence at stage \( N \) (which represents the measured data), the weights at stage \( N - 1 \), \( \{X_{j}^{N}, j = 1, ..., 2^{N-1}\} \), are obtained by simply adding the consecutive weights at stage \( N \) over non-overlapping blocks of size 2, i.e. \( X_{j}^{N} = X_{2j}^{N} + X_{2j+1}^{N} \). Associated with this stage is the time scale \( e = 2^{N-1} \). This procedure is carried out recursively. That is, given the weights of stage \( j+1 \), \( \{X_{2j}^{N-1}, j = 1, ..., 2^{j-1}\} \), we obtain the weights at stage \( j \), \( \{X_{2j}^{N-1}, j = 1, ..., 2^{j}\} \), by adding consecutive weights at stage \( j+1 \) over non-overlapping blocks of size 2, i.e.

\[
X_{2j}^{N-1} = X_{2j+1}^{N-1} + X_{2j+2}^{N-1}.
\]

for \( j = 1, ..., 2^{N-1} \). Associated with stage \( j \) is the time scale \( e = 2^{j} \). This procedure stops at stage 0, where we have a single unit weight, \( X_{1}^{0} = X_{0} \), and \( e = 2^{0} \). The latter is the largest time scale associated with the measured traffic data.

Now the multiplier distributions at different stages can be estimated as follows. From stage \( j \) to \( j+1 \), the multipliers are defined by the following equation, based on eqn. 1:

\[
P_{j}(r) \propto P_{j+1}(r) \left(1 - \frac{r}{e} \right) \}
\]

for \( j = 1, ..., 2^{N-2} \). We view \( \{P_{j}(r), i = 1, ..., 2^{j}\} \) as sampling points of the multiplier distribution at stage \( j \), \( P_{j} = P_{j}(r) 0 \leq r < e \). Subsequently, \( P_{j} \) can be determined from its histogram based on \( \{r_{j, i}^{j}, i = 1, ..., 2^{j}\} \). We then plot \( P_{j} \) against \( r \) for different stages \( j \). Together, these curves collapse together so that \( P_{j} = P_{0} \), where the multipliers distributions are stage independent, and the weights form a multifractal measure \( \mu \).

To illustrate this procedure, we analyse the first 27 video frame size data. Fig. 1 shows the obtained \( P_{j} \) distributions for \( j = 8, ..., 11 \). We observe these \( P_{j} \) curves to collapse together. This indicates that the video traffic is a multifractal process. Also note that these \( P_{j} \) curves can be well described by a truncated Gaussian:

\[
P(r) \approx e^{-\gamma_{0}(r-\gamma_{1})^{2}} 0 \leq r < e.
\]

with \( \gamma_{0} = 180 \). This function is also shown in Fig. 1 as the curve designated by asterisks.

Modelling: We model the video frame size sequence by the first 174136 weights of a multiplicative process at stage \( N = 18 \). The multiplier distribution of the process is governed by eqn. 3. We note that the parameter \( \gamma_{0} \) determines the degree of linearity of the multifractal traffic: the larger the value of \( \gamma_{0} \), the less bursty the traffic is. This property can be readily confirmed by simulation (not shown here), and can be understood if we note that when \( E_{k} \rightarrow \infty \), then the weights form a deterministic sequence, hence yielding a model that exhibits the least level of burstiness. Taking this property into account and observing that at the end of the 'Star Wars' movie there is a scene of star explosion, which contributes to

![Fig. 1 Multiplier distributions Pj, j = 8-11, for video traffic](image)

Construction of multiplicative multifractal: Consider a unit interval. Associate it with a unit mass. Divide the unit interval into two segments of equal length. Also partition the associated mass into two fractions, \( r \) and \( 1-r \), and assign them to the left and right segments, respectively. The parameter \( r \) is in general a random variable, governed by a probability density function (PDF) \( P(r), 0 \leq r \leq 1 \). The fraction \( r \) is called the multiplier. Each new interval and its associated weight are further divided into two parts following the same rule. Note that the interval length associated with stage \( j \) is \( 2^{-j} \), which corresponds to the actual time scale. We assume that \( P(r) \) is symmetric about \( r = 1/2 \), and its second moment is \( \mu_{2} \). Such a process has a number of interesting properties that render it a convenient model for LRD network traffic. For example, such a process always possesses the LRD property, with its Hurst parameter given by \( H = 1 - 1 / \log_{2}(\mu_{2}) \). A multiplicative multifractal video model refers to model the frame sizes of a video traffic by the weights of a multiplicative process at stage \( N \). When the number of the weights at stage \( N \), \( 2^{N} \), is larger than the actual number of frames \( K \), then the first \( K \) weights will be used to model the actual trace. These \( K \) weights are selected so that their mean is equal to the mean of the measured video frames. The success of the modelling of video traffic depends on determining the multiplier distribution \( P(r) \) from the measured trace data. The procedure we use is described in the following.

Analysis: Assume there are \( 2^{N} \) video frames, \( \{X_{i}, i = 1, ..., 2^{N}\} \). These frames are viewed as the weight series of a certain multiplicative process at stage \( N \). Note that the total weight \( \sum_{i} X_{i} \) is set equal to one unit. Also note that the scale associated with stage \( N \) is \( e = 2^{-N} \). This is the smallest time scale resolvable by the measured trace data.

Given the weight sequence at stage \( N \) (which represents the measured data), the weights at stage \( N - 1 \), \( \{X_{j}^{N}, j = 1, ..., 2^{N-1}\} \), are obtained by simply adding the consecutive weights at stage \( N \) over non-overlapping blocks of size 2, i.e. \( X_{j}^{N} = X_{2j}^{N} + X_{2j+1}^{N} \). Associated with this stage is the time scale \( e = 2^{N-1} \). This procedure is carried out recursively. That is, given the weights at stage \( j+1 \), \( \{X_{2j}^{N-1}, j = 1, ..., 2^{j-1}\} \), we obtain the weights at stage \( j \), \( \{X_{2j}^{N-1}, j = 1, ..., 2^{j}\} \), by adding consecutive weights at stage \( j+1 \) over non-overlapping blocks of size 2, i.e.

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large burst to the video traffic, we use 1.50 instead of the estimated value 1.80 for \( \alpha \). Fig. 2 shows a comparison of the system-size tail distributions for a single server FIFO queueing system loaded on one hand by the measured traffic (dashed lines) and on the other hand by the multifractal traffic (solid lines). The system size is represented by the queue length, which measures the total number of queued frames normalized by the average frame size. The three curves, from top to bottom, correspond to three different utilizations (i.e., normalized loading) levels, \( \rho = 0.7 \), 0.5, and 0.3. For comparison, see also the results exhibited by other more traditional models as presented in [4] for the bit loss rate. Clearly, the model presented here yields an excellent fit of the system size tail distribution for a queuing system loaded by the measured video traffic.

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References


Polyphase decision-feedback sequence estimation

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Polyphase decision-feedback sequence estimation (PDFSE) is a generalized sequence estimation algorithm that provides a tradeoff between complexity and performance in digital communications over intersymbol interference (ISI) channels. In PDFSE a sequence is divided into several polyphase components and each component is estimated separately. Interphase interference (II) is eliminated through decision-feedback. The performance/complexity ratio of the PDFSE is superior to that of reduced-state sequence estimation (RSSE) for some channels in which the energy of the channel impulse response (CIR) is not concentrated in the front part.

Introduction: Maximum-likelihood sequence estimation (MLSE) using the Viterbi algorithm (VA) is the optimal algorithm for detecting digital data over intersymbol interference (ISI) channels [1]. However, the complexity of the algorithm is proportional to the number of states of the trellis, \( M^L \), where \( M \) is the size of the signal set, and \( L \) is the channel memory length. When \( M \) and/or \( L \) are large, the algorithm becomes impractical. To reduce the complexity of the sequence estimation algorithm, delayed decision-feedback sequence estimation (DDFSE) [2], or more generally, reduced-state sequence estimation (RSSE) [3], can be used. RSSE provides a direct tradeoff between the performance and complexity of the sequence estimation algorithm. Because in the subset trellis of the RSSE, certain paths will merge earlier than in the ML trellis of the MLSE, the minimum intersubset Euclidean distance should be as large as possible, and the energy of the channel impulse response (CIR) should be concentrated in the front part as early as possible. To satisfy the second requirement, an allpass filter should be employed to make the overall CIR minimum phase. However, if the energy of the overall CIR is still not concentrated enough in the front part even if an allpass filter has been employed (such a channel will be called a poor channel in this Letter), then the performance-complexity ratio of RSSE will be poor.

To improve the performance-complexity ratio for RSSE, for poor channels, we propose a more general form of RSSE called polyphase decision-feedback sequence estimation (PDFSE) that also provides a tradeoff between the performance and complexity of the sequence estimation algorithm. In PDFSE a sequence is divided into several subsequences called polyphase components, each of which is estimated separately using RSSE. During the subsequence estimation, interphase interference (II) is eliminated through decision-feedback. In PDFSE, the number of phases \( P \) is a variable parameter. When \( P = 1 \), PDFSE degenerates into RSSE. So the performance of PDFSE is no worse than that of conventional single-phase RSSE, but for some poor channels the performance of PDFSE is better.

Channel model: The equivalent discrete-time ISI channel, which comprises a transmitting filter, a transmission media, a matched filter, a symbol-space sampler, and a discrete-time noise-whitening filter, can be modelled as a discrete-time transversal filter with its output corrupted by additive white Gaussian noise (AWGN) [1, 4]. This model can be expressed as

\[
y_n = \sum_{k=0}^{L} h_k x_{n-k} + \eta_n
\]

where \( \{x_n\} \) is the transmitted sequence, \( \{y_n\} \) is the received sequence, \( \{\eta_n\} \) is the AWGN sequence, and \( \{h_k\} \) is the overall CIR which is assumed to be minimum phase by using an allpass filter.

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