Multifractal features of sea clutter

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***Abstract*— Sea clutter refers to the backscattered returns from a patch of the sea surface illuminated by a transmitted radar pulse. Since the complicated sea clutter signals depend on the complex wave motions on the sea surface, it is reasonable to study sea clutter from nonlinear dynamics, especially chaos, point of view, instead of simply based on random processes. In the past decade, Dr. Simon Haykin's group at the McMaster University of Canada carried out analysis of some sea clutter data using chaos theory, based on the the assumption that a chaotic attractor is fully characterized by a non-integer fractal dimension and a positive Lyapunov exponent. Thus, they concluded that sea clutter signals are chaotic. In other words, the complicated sea clutter wave forms are generated by nonlinear deterministic interactions of a few modes (i.e., number of degrees of freedom). However, a numerically estimated non-integral fractal dimension and a positive Lyapunov exponent may not be sufficient indication of chaos. Recently, Cowper and Mulgrew, Noga, and Davies separately have questioned the chaoticness of the radar sea clutters. In this paper, we show, using the direct dynamical test for deterministic chaos developed by Gao and Zheng, which is one of the more stringent criteria for low-dimensional chaos, a two minute duration sea clutter data is not chaotic. We also carry out a multifractal analysis of this sea clutter data set, and find that the original sea clutter amplitude signal is approximately multifractal, while the envelope signal, formed by picking up the successive local maxima of the amplitude signal, thus measuring the energy of successive waves on the sea surface, is well modeled as multifractals. A possible interpretation for this difference is that when time scales are short enough to represent the detailed signal corresponding to the individual wave motion turning over on the sea surface, the multifractal scaling breaks. These behaviors determine that the amplitude signal follows approximately log-normal distribution while the envelope signal, and thus the energy of the successive waves of the sea surface, is log-normally distributed. Approximate log-normal distributions for the amplitude signal has been observed earlier. However, by using the multiplicative multifractal theory, there is theoretical justification for the log-normal distribution of sea clutter, as discussed in the manuscript. The implications of the multifractal nature of sea clutter may have relevance for the detection of point targets on the sea surface.**

I. Introduction

Sea clutter refers to the backscattered returns from a patch of the sea surface illuminated by a transmitted radar pulse. The study of sea clutter is not only of theoretical importance, but may also be of practical relevance. For example, it may help us understand the severe limitations on the detectability of radar returns from "point" targets such as low-flying aircraft, small marine vessels, navigation buoys, small pieces of ice, etc. Hence, understanding the true nature of sea clutter is an important and challenging problem.

Sea clutter waveforms are generally quite complicated. Traditionally sea clutter is often studied in terms of certain simple statistical features, such as the marginal probability density function. For example, in the phenomenological modeling of sea clutter, it has been recognized for quite a while that sea clutter amplitude signals are approximately log-normally distributed [14]. That is, in a logarithmic scale, the amplitude signals are approximately normally distributed. Such modeling, however, basically does not shed much light on the true nature of sea clutter. Hence, one does not have any insight as how good the log-normal distribution is for the sea clutter signals, and where is the basis for this log-normality in the sea clutter data.

Since the complicated sea clutter signals are functions of complex (sometimes turbulent) wave motions on the sea surface, while wave motions on the sea surface clearly have their own dynamical features that are not readily described by simple statistical features, it is thus very desirable to understand sea clutter by considering some of their dynamical features. To this end, a natural question to ask is whether sea clutter is low-dimensional deterministic chaos. That is, if the complicated sea clutter wave forms are generated by nonlinear deterministic interactions of a few modes (i.e., number of degrees of freedom). If the answer is yes, then the apparent randomness in the sea clutter signals has a deterministic origin. This would contrast sharply with the modeling of sea clutter based solely on random processes.

Chaos is also commonly called a strange attractor. Being an "attractor" means the trajectories in the phase space are bounded. Being "strange", the nearby trajectories, on the average, diverge exponentially fast. Mathematically, the latter property can be expressed as follows. Let $d(0)$ be the small separation between two arbitrary trajectories at time 0, and $d(t)$ be the separation between them at time t. Then, for true low-dimensional deterministic chaos, we have

$$d(t) \sim d(0)e^{\lambda_1 t}, \tag{1}$$

where λ_1 is positive and called the largest Lyapunov exponent. Due to the boundedness of the attractor and the exponential divergence between nearby trajectories, a strange attractor typically is a fractal, characterized by a simple and elegant scaling law:

$$N(\varepsilon) \sim \varepsilon^{-D} \quad (2)$$

where N represents the (maximal) number of boxes, of linear length not larger than ε, needed to cover the attractor, and D is typically a non-integral number called the fractal dimension of the attractor. A non-integral fractal dimension contrasts sharply with the integer-valued topological dimension (which is 0 for finite number of isolated points, 1 for a smooth curve, 2 for a smooth surface, and so on).

In the past decade, Dr. Simon Haykin's group at the McMaster University of Canada has carried out analysis of some sea clutter data using chaos theory [11], [12], based on the assumption that strange attractors are fully characterized by a non-integral fractal dimension and a positive Lyapunov exponent. By numerical computations, they found a non-integral correlation dimension (which is a tight lower bound for the fractal dimension [10]) and a positive largest Lyapunov exponent from their data. Thus they suggested that sea clutter be modeled by low-dimensional strange attractors.

However, a non-integral dimension together with a positive largest Lyapunov exponent obtained by computational means instead of analysis may not be a sufficient indication of deterministic chaos. A well-known example is the so-called $1/f$ processes. These are random processes with spectral density

$$S(f) \sim f^{-(2H+1)}, \quad (3)$$

where $0 < H < 1$ is sometimes called the Hurst parameter. A trajectory formed by such a process has a fractal dimension of $1/H$. As we shall explain shortly, with most algorithms of estimating the largest Lyapunov exponent, one obtains a positive number for the "Lyapunov exponent", and interpret the random process as originating from deterministic chaos.

Recently, Cowper and Mulgrew [1], [2] , Noga [15] and Davies [3] separately have questioned the chaoticness of the radar sea clutters, by comparing the similarities/differences between sea clutter signals and certain well studied chaotic systems and random processes. In particular, Cowper and Mulgrew [2] claimed "If sea clutter is chaotic, then nonlinear predictors will be able to exploit this property." From both a wave tank data set and two radar clutter data sets collected at sea, they showed no evidence of nonlinear predictability.

Since the approaches of Cowper and Mulgrew [1], [2] , Noga [15] and and Davies [3] are indirect, it would be very desirable to directly re-assess if sea clutter is really chaotic. If the answer is yes, then one can specifically ask how may log-normal distribution and chaos coexist for sea clutter. To answer these questions, in this paper we employ a number of signal processing tools from the more advanced chaos and fractal theories to analyze a sea clutter dataset measured on a patch of free sea surface (i.e., without point targets), supplied by Dr. Simon Haykin's group. We shall show that sea clutter signals (at least for this data set) are not low-dimensional chaos, but are multifractals. As we will show, the multifractal nature of sea clutter justifies why the sea clutter signals are log-normally distributed.

The paper is organized as follows. In Sec. 2, we briefly describe the data. In Sec. 3, we study the sea clutter data employing the direct dynamical test for low-dimensional chaos developed by Gao and Zheng [7], and show that for this data set, the sea clutter is not chaotic. In Sec. 4, we carry out a multifractal analysis of the sea clutter signals. Sec. 5 contains some concluding remarks.

II. The sea clutter data

The sea clutter data analyzed here is supplied by Dr. Simon Haykin's group. The sampling rate is 1000 Hz. The data set is 2^{17} points long. Fig. 1 shows two short segments of the amplitude signal. We notice that the signal between two successive local maxima is about 10 points long and are reasonably smooth. The waveform between two successive local maxima may be considered to correspond to the gradual turning over of a wave on the sea surface. To have a feeling of how the entire amplitude signal looks like, we have plotted it in Fig. 2(a). Now we notice that the signal is very irregular. Fig. 2(b) has plotted the envelope signal of the data, which is formed by picking up the successive local maxima of the data. Local maxima of the amplitude of the data may be thought of as the square root of the energy of the successive waves of the sea surface. Thus, the irregularity of the data reflects the wide variation of the energy of successive waves turning over on the sea surface. We shall show below that log-normal distributions are only applicable to the envelope signal, not to the original amplitude data. In other words, the energy of successive waves on the sea surface follows log-normal distribution. This simply follows from that if x follows log-normal distribution, then x^2 also follows log-normal distribution.

Finally, we note that even though we only analyze one set of sea clutter signal in this paper, the many sea clutter signals analyzed by Dr. Haykin's group, Cowper and Mulgrew [1], [2], Noga [15], and Davies [3] look quite similar to ours, hence, we expect the results presented in our paper are very likely generic.

III. Are sea clutter signals chaotic?

In this section, to study the sea clutter data set, we employ the direct dynamical test for low-dimensional chaos developed by Gao and Zheng [7], which is one of the more stringent test for chaos available in the literature.

Given a scalar time series, $x(1), x(2), ..., x(N)$, (assuming, for convenience, that they have been normalized to the unit interval [0,1]), one first constructs vectors of the following form using the time delay embedding technique [18]: $V_i = [x(i), x(i+L), ..., x(i+(m-1)L)]$, with m being the embedding dimension and L the delay time. For example, when $m = 3$ and $L = 4$, we have $V_1 = [x(1), x(5), x(9)]$, $V_2 = [x(2), x(6), x(10)]$, and so on. For the analysis of purely chaotic signals, m and L

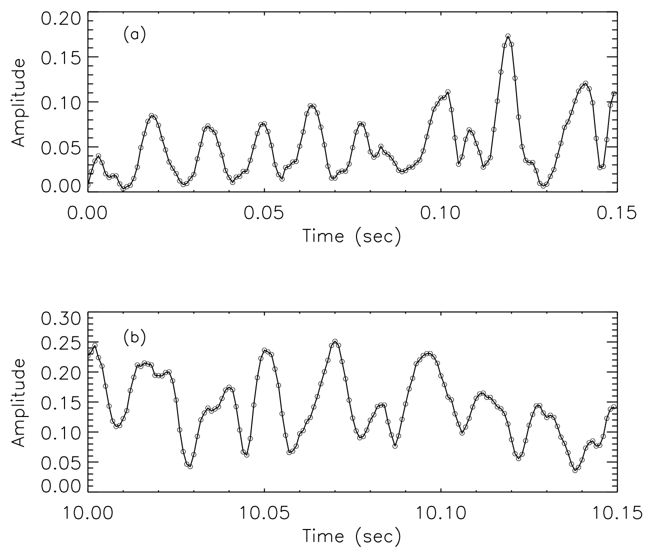

Fig. 1. Two short segments of the amplitude sugnal of the sea clutter data.

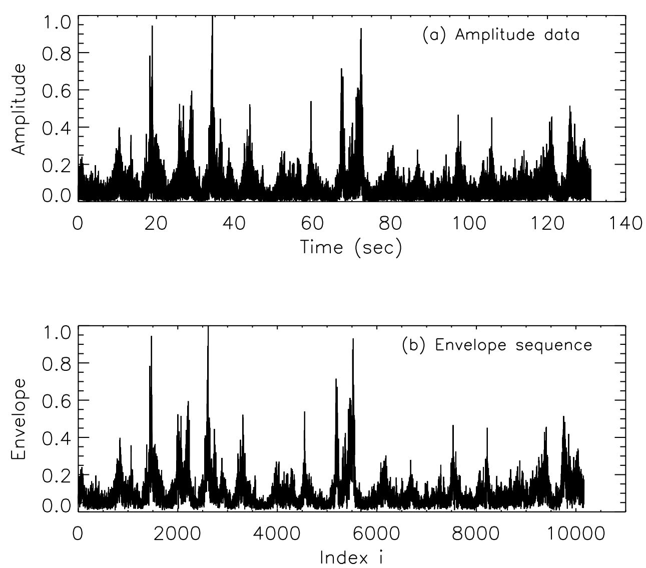

Fig. 2. The complete amplitude signal (a) and its envelope (b).

has to be chosen properly. This is the issue of optimal embedding (see [6], [7] and references therein). After the scalar time series is properly embedded, one then computes

$$\Lambda(k) = \left\langle \ln\left(\frac{||V_{i+k} - V_{j+k}||}{||V_i - V_j||}\right)\right\rangle \tag{4}$$

with $r \le ||V_i - V_j|| \le r + \Delta r$, where r and Δr are prescribed small distances. The angle brackets denote ensemble averages of all possible pairs of (V_i, V_j) and k is called the evolution time. A pair of r and Δr is called a shell. The computation is typically carried out for a sequence of shells. (One should compare Eq. (4) with Eq. (1) to gain a better understanding.) For true low-dimensional chaotic systems, the curves $\Lambda(k)$ vs. k for different shells form a common envelope. The slope of the

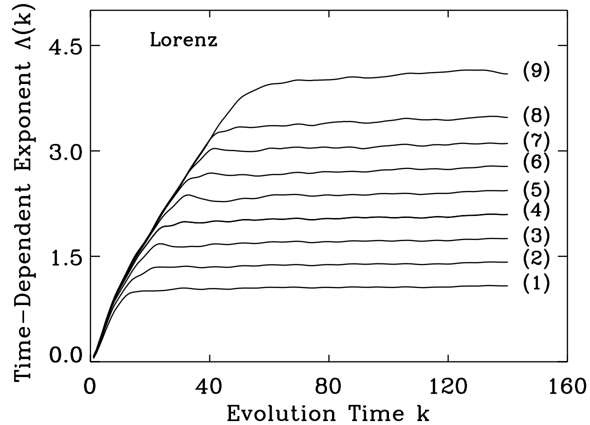

Fig. 3. $\Lambda(k)$ curves for the chaotic Lorenz system. Reproduced from Fig. 7(a) of [7].

envelope estimates the largest positive Lyapunov exponent. An example is given in Fig. 3 for the well-known chaotic Lorenz system. We note the common envelope at the lower left corner of Fig. 3. The existence of that common envelope guarantees that a robust positive Lyapunov exponent will be obtained by different researchers no matter which shell they use in the computation, thus ensures determinism. For non-chaotic systems, the common envelope is absent. As an example, Fig. 4 shows the $\Lambda(k)$ vs. k curves for a set of uniformly distributed random variables. Here, we note there is no common envelope in the lower left corner of Fig. 4. We also note, most other algorithms for estimating the largest Lyapunov exponent is equivalent to compute $\Lambda(k)$ for $r < r_0$, where r_0 is selected more or less arbitrarily, then obtain $\Lambda(k)/k$, for not too large k, as an estimation of the largest Lyapunov exponent. With such algorithms, one can obtain a "positive" Lyapunov exponent for white noises and for $1/f$ processes. However, this positive number critically depends on the parameter r_0 that is selected to use in the computation, hence, typically is different for different researchers. We thus observe a random element here! While there is little danger to interpret white noise as low-dimensional deterministic chaos (as its dimension equals the embedding dimension for very long time series, thus not a non-integer), the possibility of mistaking $1/f$ processes as "chaotic" may result from using certain computational algorithms that inappropriately yield "positive" Lyapunov exponent.

The $\Lambda(k)$ vs. k curves for the sea clutter signal is shown in Fig. 5, with $m = 7$ and $L = 2$. Similar curves were obtained for another choices of M and L. We do not observe a common envelope here. Hence, we have to conclude that for the given sea clutter data set, it is not chaotic.

IV. MULTIFRACTAL ANALYSIS OF SEA CLUTTER

To gain deeper insights from sea clutter data, in this section we perform a multifractal analysis of sea clutter signals. Mathematically, multifractals are characterized by many or infinitely many power-law relations. Strange attractors, as ge-

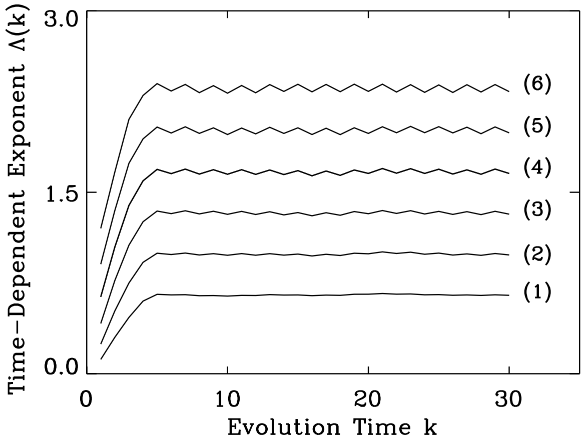

Fig. 4. $\Lambda(k)$ curves for uniformly distributed random noise. Reproduced from the left panel of Fig. 8 of [7]

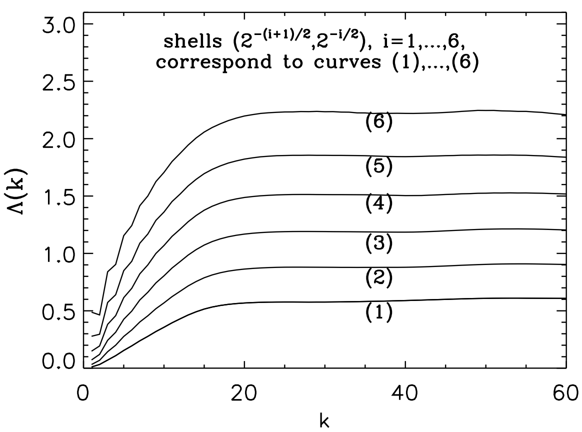

Fig. 5. $\Lambda(k)$ curves for sea clutter.

ometrical objects, are often multifractals, in the sense that, the entire attractor may be partitioned into many (possibly infinitely many) subsets, each subset has its own fractal dimension, and its "weight" in the original attractor is well defined. In this paper, we work with a specific type of multifractal, called random multiplicative process model, to analyze sea clutter. Such models have been used to describe the energy dissipation of turbulence [5], rain fall [16], amount of radiation the earth received from the Sun [4], stock variation [13], and network traffic [8], [9].

In this framework, one computes

$$M_q(\varepsilon) = \sum_i w_i^q, \tag{5}$$

where q is a real number, and the positive "weights" w_i, $i = 1, 2, \ldots$, can be readily computed from a time series, as we shall explain shortly. One then checks if the following scaling law holds,

$$M_q(\varepsilon) \sim \varepsilon^{\tau(q)}. \tag{6}$$

If Eq. (6) holds and $\tau(q)$ is not a linear function of q, then we

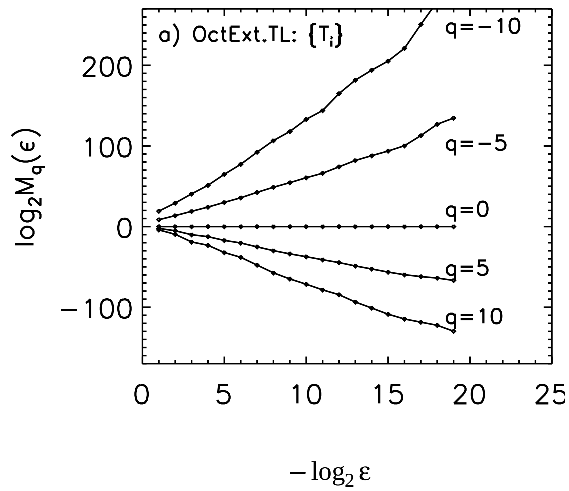

Fig. 6. $\log_2 M_q(\varepsilon)$ vs. $-\log_2 \varepsilon$ for the interarrival time series of WAN traffic. Reproduced from Fig.8(a) of [8].

conclude the waveform is a multifractal. Note that large positive q "emphasizes" large weights, while large negative q "emphasizes" small weights. Also note that $\tau(q)$ being nonlinear in q is equivalent to the weights w_i begin not constants. Thus, when w_i are obviously nonuniform, one only needs to check if the scaling described by Eq. (6) holds or not. An example of a multifractal time-series is shown in Fig. 6 for some network traffic data [8].

To better understand multifractal formalism, we consider below the random multiplicative cascade model. The construction rule is as follows: Consider a unit interval. Associate it with a unit mass. Divide the unit interval into two (say, left and right) segments of equal length. Also partition the associated mass into two fractions, r and $1-r$, and assign them to the left and right segments, respectively. The parameter r is in general a random variable, governed by a probability density function (pdf) $P(r)$, $0 \le r \le 1$. The fraction r is called the multiplier. Each new subinterval and its associated weight are further divided into two parts following the same rule. This procedure is schematically shown in Fig. 7, where the multiplier r is written as r_{ij}, with i indicating the stage number, and j assuming odd integers designating the location of each weight at that stage (leaving the positions corresponding to even integers to $1-r_{ij}$). Note the scale (i.e., the interval length) associated with stage i is 2^{-i}. We assume that $P(r)$ is symmetric about $r = 1/2$, and has successive moments $\mu_1, \mu_2, \ldots$. Hence r_{ij} and $1-r_{ij}$ both have marginal distribution $P(r)$. The weights at the stage N, $\{w_n, n = 1, \ldots, 2^N\}$, can thus be expressed as $w_n = u_1 u_2 \cdots u_N$, where u_l, $l = 1, \ldots, N$, are either r_{ij} or $1-r_{ij}$, hence, $\{u_i, i \ge 1\}$ are independent identically distributed (i.i.d) random variables (RVs) having pdf $P(r)$. Being a multiplication of a sequence of i.i.d RVs, the weights w_n thus have a log-normal distribution. It can be easily shown that Eq. (6) holds for such processes, with $\tau(q) = -\ln(2\mu_q)/\ln 2$ [8].

We note that when the above processes are used to model

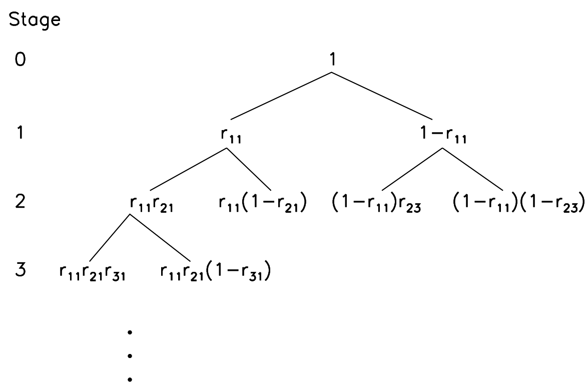

Fig. 7. Schematic of the construction rule.

the aggregate (i.e., counting) network traffic processes, the unit time interval is the total time span of interest. Let us say it is 1 year. The unit mass (or weight) is the total traffic loading to a link. Then, r_{11} and $1-r_{11}$ denotes the traffic loading to the link in the first and second half year, respectively. Since typically $r_{11} \neq 1/2$, we thus see that the traffic loading to the link is different for the two half years. Such feature cannot be considered as stationary. While one might expect that traffic loading to a network will eventually be almost a constant when the time interval one considers is long enough, such "nice" behavior actually has not been observed [17]. Perplexing it may be, counter-intuition sometimes just prevails.

Let us now explain how to obtain the weights $w_i,\ i = 1, 2, ...,$ from a positive time series such as the sea clutter amplitude signals. Assume we have a positive time series, $\{X_i, i = 1, \cdots, 2^N\}$. We interpret them as the weight sequence at stage N. The weights at stage N−1, $\{X_i^{(2^1)}, i = 1, \cdots, 2^{N-1}\}$, can be obtained by simply adding the consecutive weights at stage N over non-overlapping blocks of size 2, i.e., $X_i^{(2^1)} = X_{2i-1} + X_{2i}$, for $i = 1, \cdots, 2^{N-1}$, where the superscript 2^1 for $X_i^{(2^1)}$ is used to indicate that the block size used for the involved summation at stage N−1 is 2^1. Associated with this stage is the scale $\varepsilon = 2^{-(N-1)}$. This procedure is carried out recursively. That is, given the weights at stage j+1, $\{X_i^{(2^{N-j-1})}, i = 1, \cdots, 2^{j+1}\}$, we obtain the weights at stage j, $\{X_i^{(2^{N-j})}, i = 1, \cdots, 2^j\}$, by adding consecutive weights at stage j+1 over non-overlapping blocks of size 2, i.e.,

$$X_i^{(2^{N-j})} = X_{2i-1}^{(2^{N-j-1})} + X_{2i}^{(2^{N-j-1})} \quad (7)$$

for $i = 1, \cdots, 2^j$. Here the superscript 2^{N-j} for $X_i^{(2^{N-j})}$ is used to indicate that the weights at stage j can be equivalently obtained by adding consecutive weights at stage N over non-overlapping blocks of size 2^{N-j}. Associated with stage j is the scale $\varepsilon = 2^{-j}$. This procedure stops at stage 0, where we have a single unit weight, $\sum_{i=1}^{2^N} X_i$, and $\varepsilon = 2^0$. The latter is the largest time scale associated with the measured data. Fig. 8 schematically shows this procedure.

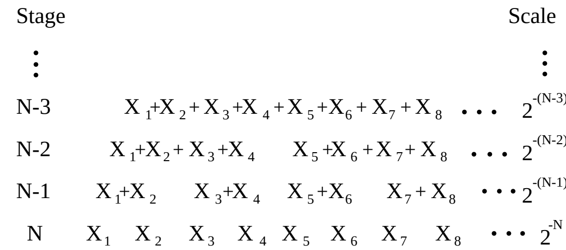

Fig. 8. Schematic for obtaining weights from a time series.

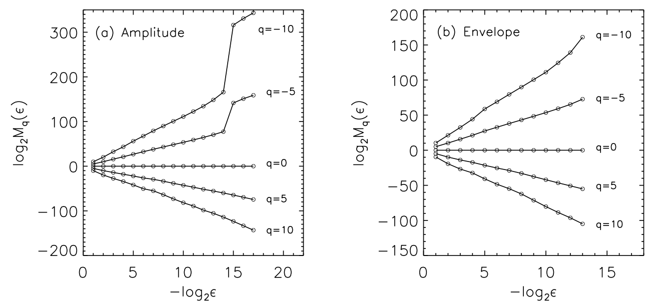

Fig. 9. $\log_2 M_q(\varepsilon)$ vs. $-\log_2 \varepsilon$ for the amplitude and envelope signals of sea clutter.

Fig. 9(a) shows the result of multifractal analysis on the amplitude signal of sea clutter. We observe that the multifractal scaling is valid for all positive q as well as for negative q on not too short time scales. Since negative q emphasizes the part of the signal with small magnitudes, the above feature thus suggests that the short portion of the data between successive local maxima does not follow this multifractal scaling laws. We are therefore motivated to carry out multifractal analysis on the envelope signal of Fig. 2(b). The result is shown in Fig. 9(b). Indeed, now we see the multifractla scaling holds for all q on all time scales.

The results of Fig. 9 suggest that the amplitude signals will only be approximately log-normally distributed, while the envelope signal will be more so. This is indeed the case, as shown in Fig. 10(a) and (b). This simply suggests that the detailed signals for each wave motion on the sea surface, represented by the data between successive local maxima, do not follow log-normal distributions. Of course, intuitively this makes total sense.

V. Concluding Remarks

It is well known that the measurement and characterizing of radar sea-clutter returns are quite complicated. Since the observed clutter waveforms are the output of a complicated system with the input being the radar pulses, it is important to have as much as possible various parameters of the input and the system. Such system and physical parameters may include: RF wavelength, PRF, IF band width, video bandwidth, sea patch extent, grazing angle, sea state, ocean depth, predetection noise level, A/D quantization value, etc. Some of

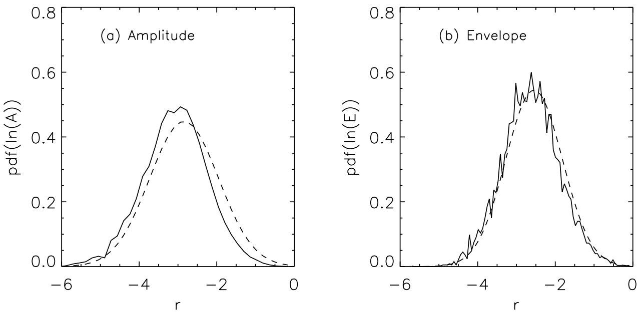

Fig. 10. Distributions of amplitude and envelope signals (in logarithmic scale) for sea clutter. Dashed lines are normal distributions.

these system parameters should be available from the experimental designers while other may need to be measured by the experimentalists who have performed the data measurements. In practice, obtaining these parameters may not be trivial. Furthermore, in order to make any definitive conclusion about the nature of sea-clutters, extensive collection under various conditions must be performed.

In this paper, our goal is more limited since we have only one two-minute set of sea-clutter data. By employing some more advanced tools for signal processing from chaos and fractal theory, we have found that (at least for this) sea clutter data set, it is multifractal but not chaotic. The multifractal scaling laws hold well for the envelope signal of the sea clutter amplitude data, but breaks for the original amplitude signal on time scales small enough to represent individual waves on the sea surface (which is "emphasized" by negative q). This determines that log-normal distributions describe quite well the envelope signal while only approximately the amplitude signal. We note that such finding may have profound practical implications. For example, the multifractal nature of free sea surface clutter may be quite different from that of sea clutter with point targets. Two simple scenarios for such possibility would be that with a point target, the multifractal scaling is completely absent, or profoundly modified. In either case, a multifractal theory may be able to help us to develop better algorithms to detect point targets on the sea surface.

Acknowledgments

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