Multiplicative multifractal modeling of sea clutter

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ABSTRACT

Sea clutter refers to the radar returns from a patch of ocean surface. Accurate modeling of sea clutter and robust detection of low observable targets within sea clutter are important problems in remote sensing and radar signal processing applications. In this paper, we introduce a multiplicative multifractal process for modeling the sea clutter. Through analysis of 392 amplitude time series of 14 sea clutter datasets measured under various sea and weather conditions, we show that the data are consistent with multifractals over certain time scale range, and that the generalized dimension spectrum D_q accurately detects low observable targets within sea clutter. The method is computationally very fast and practically easily implemented. These attributes strongly suggest that the method may be developed into an automated target detector within sea clutter.

1. *INTRODUCTION*

Sea clutter is the backscattered returns from a patch of the sea surface illuminated by a radar pulse. Robust detection of low observable targets within sea clutter has significant importance to our coastal and national security, since objects within sea clutter may include submarine periscopes, low-flying aircrafts, and missiles. The task is also very important to navigation safety and environmental monitoring, since objects within sea clutter can be small marine vessels, navigation buoys, small pieces of ice, patches of spilled oil, etc. Accurate modeling of sea clutter is an important problem in remote sensing and radar signal processing applications. Sea clutter study may also help understand the propagation of electromagnetic waves so that wireless communication channel characterization and signal detection can be greatly improved.

Due to the multitude of waves and the almost infinite reflecting angles they present, sea clutter is often highly non-Gaussian, especially in heavy sea conditions. Hence, sea clutter modeling is a very difficult problem, and a lot of effort has been made to study sea clutter. Traditionally sea clutter is often studied in terms of certain simple statistical features, such as the marginal probability density function. The non-Gaussian feature of sea clutter has motivated researchers to employ weibull, log-normal, Compound-K and Compound-Gaussian distributions to model sea clutter. Such simple phenomenological modeling of sea clutter does not help one gain much analytical or physical understanding, however.

To gain deeper understanding of the nature of sea clutter, the concept of fractal has been employed for the description [1, 2] and modeling [3 - 6] of the roughness of sea surface, and investigation of scattering from rough surfaces modeled by fractal processes [7 – 10]. Possible chaotic behavior of sea clutter has also been studied [11, 12].

Since the ultimate goal of sea clutter study is to improve the detection of targets embedded within the sea clutter, a lot of effort has been made to design innovative methods for target detection within sea clutter. Notable approaches include extinction-pulse techniques, time-frequency analysis techniques, wavelet based approaches, neural network based approaches and wavelet-neural net combined approaches, as well as utilizing the concept of fractal dimension [13] and fractal error [14, 15], and boxing-counting based multifractal analysis [16]. It appears that the most thoroughly evaluated scheme is the one based on a neural network [20]. Note that most of the above works were based on the analysis of radar images. To improve detection accuracy, some researchers resort to more powerful millimeter wave radars with higher resolution. The status of the field clearly indicates that one needs to adopt a systematic approach, work on a large number of datasets measured under various sea and weather conditions, and design a few readily computable parameters that can accurately and easily detect targets within sea clutter.

In this paper, we introduce a fractal model, called mulitplicative cascade multifractal, for the modeling of sea clutter amplitude data, with the purpose of helping distinguish time series with and without targets.

The remaining of the paper is organized as follows. In section 2, we briefly describe the sea clutter data. In section 3, we first overview measure based multifractal, then, we briefly describe multiplicative cascade multifractal. In section 4, we first describe a general procedure to carry out multifractal analysis of experimental data, then, we perform a detailed analysis of sea clutter data. We show that the sea clutter exhibits stochastic features which are consistent with the multiplicative cascade multifractal model. In particular, we show that the model appears to be able to detect small objects in sea clutter. We summarize our findings in the final section.

2. *SEA CLUTTER DATA*

We have obtained 14 sea clutter datasets from a website maintained by Professor Simon Haykin: http://soma.ece.mcmaster.ca/ipix/dartmouth/datasets.html. The measurement was made with an IPIX radar of RF 9.39 GHz (and hence a wavelength of about 3 cm). The grazing angle varied from less than 1 degree to a few degrees. The wave height in the ocean varied from less than 0.8m to 3.8m (with peak height up to 5.5m). The wind conditions varied from still to 60 km/hr (with gusts up to 90 km/hr). We analyze two polarizations, HH (horizontal transmission, horizontal reception) and VV (vertical transmission, vertical reception). Each dataset contains 14 spatial range bins of HH as well as 14 range bins of VV time series. Therefore, there are a total of 392 time series. A few of the range bins hit a target, which was made of a spherical block of styrofoam of diameter 1 m, wrapped with wire mesh. The range bins where the targets are strongest are called primary bins, while neighboring range bins where the targets may also be visible are called secondary bins. Each range bin data contains 2^{17} complex numbers, with a sampling frequency of 1000 Hz. We analyze the amplitude data. Two typical time series are shown in Figs. 1(a, b).

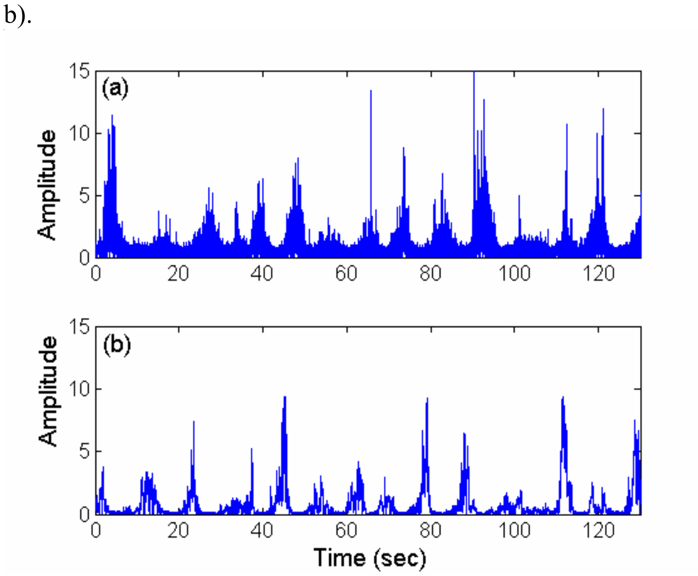

Fig.1 Sea clutter amplitude data from the same dataset. (a) range bin without target, (b) range bin with primary target

3. *MULTIPLICATIVE CASCADE MULTIFRACTALS*

In this section, we first overview the definition of the measure based multifractals. This type of multifractals is typically constructed through multiplicative processes.

3.1 Definition

Consider a unit mass with unit interval, and partition the mass into a series of small intervals, each of linear length ε. Also partition the unit mass into a series of weights or probabilities $\{w_i\}$, and associate the weight w_i with the ith interval. Now consider the moments

$$M_q(\varepsilon) = \sum_i w_i^q , \qquad (1)$$

where q is real. Note the convention that whenever w_i is zero, the term w_i^q is dropped. We also note that a positive q value emphasizes large weights, while a negative q value emphasizes small weights. If we have a real function $\tau(q)$ of q,

$$M_q(\varepsilon) \sim \varepsilon^{\tau(q)}, as \ \varepsilon \to 0 \qquad (2)$$

for every q, and the weights $\{w_i\}$ are non-uniform, then the weights $w_i(\varepsilon)$ are said to form a multifractal measure. Note that the normalization $\sum_i w_i = 1$ implies that $\tau(1) = 0$. Note that if $\{w_i\}$ are uniform, then $\tau(q)$ is linear in q. When $\{w_i\}$ is weakly non-uniform, visually $\tau(q)$ may still be approximately linear in q. The non-uniformity in $\{w_i\}$ is better characterized by the so called generalized dimensions D_q, defined as [18]

$$D_q = \frac{\tau(q)}{q-1}, \qquad (3)$$

D_q is a monotonically decreasing function of q [17]. It exhibits a nontrivial dependence on q when the weights $\{w_i\}$ are non-uniform.

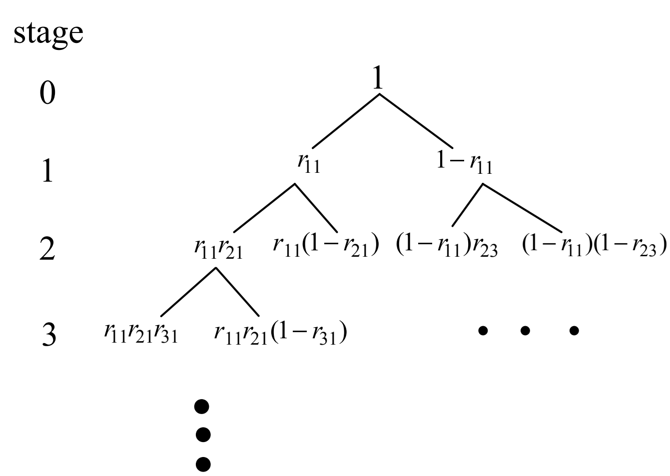

Fig.2 Schematic illustrating the construction rule of a multiplicative multifractal.

3.2 Construction of multiplicative multifractals

To better appreciate the construction rules, we point out that these rules essentially involve dyadic partitions.

Consider a unit mass with a unit interval. Divide the mass into two segments of equal length, with associated mass r and 1-r, and assign them to the left and right segments, respectively. The parameter r is in general a random variable, governed by a probability density function (pdf) $P(r), 0 \le r \le 1$. The fraction r is called the multiplier. Each new subinterval with associated weight is further divided into two parts following the same rule. This procedure is schematically shown in Fig. 2, where the multiplier r is written as r_{ij}, with i indicating the stage number, and j (assuming only odd numbers thus leaving even numbers for $1-r_{ij}$) indicating the positions of a weight on that stage. Note the scale (i.e., the interval length) associated with stage i is 2^{-i}. We assume that $P(r)$ is symmetric about $r=1/2$, and has successive moments $\mu_1, \mu_2, \cdots$ Hence r_{ij} and $1-r_{ij}$ both have marginal distribution $P(r)$. The weights at the stage N, $\{w_n, n=1,\cdots,2^N\}$, can be expressed as $w_n = u_1 u_2 \cdots u_N$, where $u_l, l=1,\cdots,N$, are either r_{ij} or $1-r_{ij}$, thus, $\{u_i, i \ge 1\}$ are independent identically distributed random variables having pdf $P(r)$. The function $P(r)$ can be selected to follow any functional form [17].

To appreciate the effectiveness of the cascade process in modeling sea clutter data, Figs.3(a, b) show two simulated time series, one intended to model the measured data of Fig.1 (a), the other for Fig.1 (b). We observe that the model is quite good.

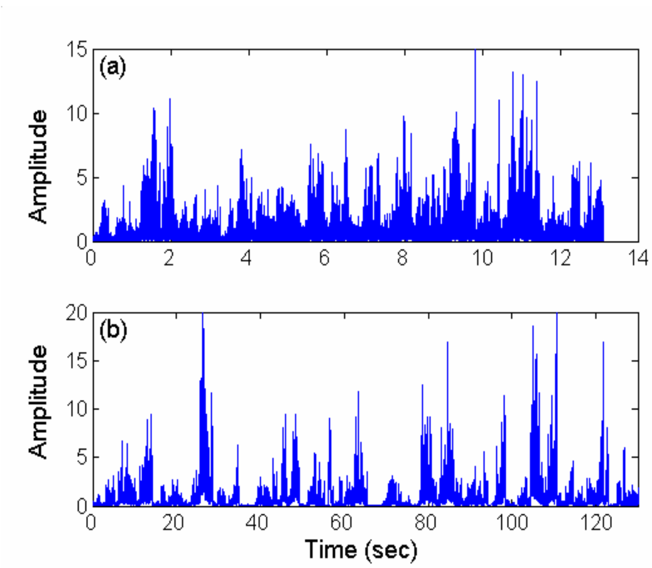

Fig.3 Simulated time series using multiplicative process. (a, b) Simulating sea clutter data in Fig. 1(a, b) respectively.

3.3 Why cascade processes are multifractals

To show that cascade processes are multifractals, let us prove the following property for the weights at stage N [19]. Theorem:

$M_q(\varepsilon) \sim \varepsilon^{\tau(q)}$ with $\varepsilon = 2^{-N}, \tau(q) = -\ln(2\mu_q)/\ln 2$.

This follows the observation that at stage N,

$$M_q(\varepsilon) \approx 2^N E(w^q) = 2^N E((u_1 \cdots u_N)^q) = 2^N \mu_q^N.$$

The property indicates that a multiplicative process is a multifractal, and relates the $\tau(q)$ spectrum to the moments of the multiplier distribution. One of the simplest examples is random binomial multiplicative process, for which the multiplier distribution is given by $P(r) = [\delta(r-p) + \delta(r-(q-p))]/2$, where $\delta(x)$ is Kronecker delta function. For this process, we have $\mu_q = [p^q + (1-p)^q]/2$, thus, $\tau(q) = -\ln[p^q + (1-p)^q]/\ln 2$.

4. *MULTIPLICATIVE MULTIFRACTAL ANALYSIS OF SEA CLUTTER*

In this section, we show that the sea clutter data exhibit stochastic features consistent with the stochastic behavior of random multiplicative processes over certain time scale range. We shall further show that these features can be used to detect targets in sea clutter.

Let us first describe the general procedure for computing the moments $M_q(\varepsilon)$ defined in Eq. (1) at different stages, and checking whether Eq. (2) is valid for certain ε range. Assuming we use 2^N consecutive amplitude data. For ease of illustration, we denote the square of raw data by $\{x_i\}$. We view $\{x_i, i=1,2\cdots 2^N\}$ as the weight series of a certain multiplicative process at stage N. Note that the total weight $\sum_1^{2^N} x_i$ is set equal to 1 unit. Also note the scale associated with stage N is $\varepsilon = 2^{-N}$. This is the smallest time scale resolvable by the measured data. Given the weights at stage j+1, $\{X_i^{(2^{N-j-1})}, i=1,\cdots,2^{j+1}\}$, we obtain the weights at stage j, $\{X_i^{(2^{N-j})}, i=1,\cdots,2^j\}$, by adding consecutive weights at stage j+1 over non-overlapping blocks of size 2.

$$X_i^{(2^{N-j})} = X_{2i-1}^{(2^{N-j-1})} + X_{2i}^{(2^{N-j-1})}$$

for $i=1,\cdots,2^j$. Here the superscript 2^{N-j} for $X_i^{(2^{N-j})}$ is used to indicate that the weights at stage j can be equivalently obtained by adding consecutive weights at stage N over non-overlapping blocks of size 2^{N-j}. The scale associated with stage j is $\varepsilon = 2^{-j}$.

After we have obtained all the weights from stage 1 to N, we compute the moments $M_q(\varepsilon)$ according to Eq. (1) for different values of q. We then plot $\log M_q(\varepsilon)$ *vs.* $\log \varepsilon$ for different values of q. If these curves are linear over wide range of ε, then the weights are consistent with a multifractal measure. Note that, according to Eq. (2), the slopes of the linear part of $\log M_q(\varepsilon)$ *vs.* $\log \varepsilon$ curves provide an estimate of $\tau(q)$, for different values of q.

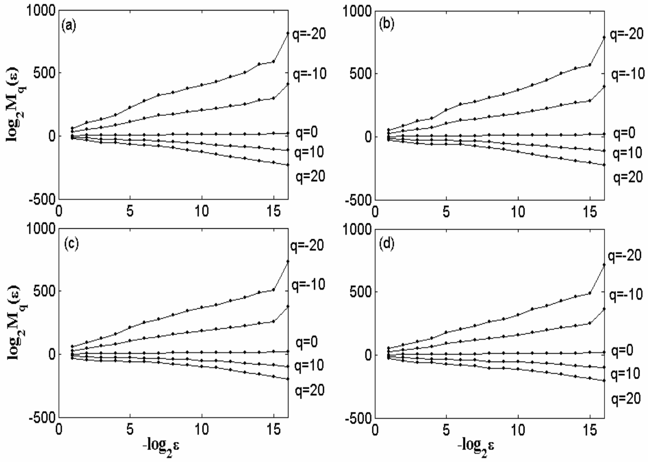

Fig.4 $\log M_q(\varepsilon)$ *vs.* $\log \varepsilon$ curves of sea clutter amplitude data in the same measurement. (a) HH data with primary target; (b) VV data with primary target; (c) HH data without target; (d) VV data without target.

We illustrate the above procedure by using all 2^{17} points of each sea clutter dataset for this analysis, which is around 130 seconds. Figs. 4(a - d) show representative $\log M_q(\varepsilon)$ *vs.* $\log \varepsilon$ curves for (a) a HH polarization data with primary target, (b) VV data with primary target, (c) HH data without target, and (d) VV data without target. We observe that the degree of linearity between $\log M_q(\varepsilon)$ and $\log \varepsilon$ for all data sets is quite good up to the 15^{th} stage.

To further check whether these data sets are truly multifractals, we compute D_q for certain ε range. Here we focus on the range from stage 6 to stage 12, corresponding to the time scale range of 32ms to about 2s, which especially exhibits the difference of the structure features between range bins with and without targets. Note here we mainly focus on large weights, which are emphasized by positive q. Representative results of D_q are shown in Fig.5. Indeed, we observe that in all cases D_q has a nontrivial dependence on q. Therefore, we conclude that these time series are consistent with multifractals. More important, D_q with a big positive value can distinguish the range bin with target. For example, when $q = 30$, the D_q values given by the range bins with targets are much closer to 1 than those given by the range bins without targets.

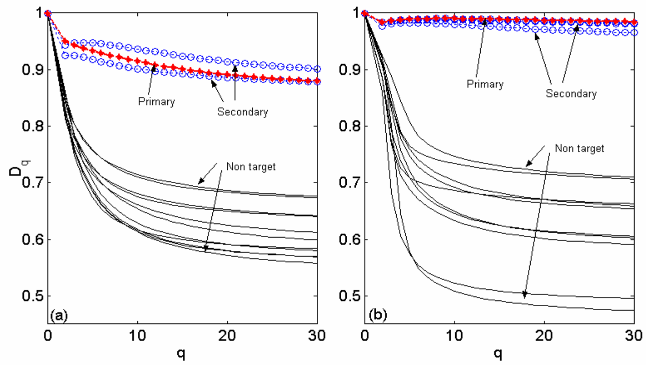

Fig.5 The generalized dimension spectrum
(a) HH and (b) VV amplitude dataset.

We have systematically studied all 392 time series of 14 sea clutter datasets. To better appreciate the detection performance, we have first focused on bins with primary targets, but omitted those with secondary targets, since sometimes it is hard to determine whether a bin with secondary target really hits a target or not. After omitting the range bins with secondary targets, the frequencies for the $D_q\,(q = 30)$ under the two hypotheses (the bins without targets and those with primary targets) for VV datasets are shown in Fig.6. We observe that the accuracy for the detection is very good, except for one primary bin. Careful examination of the amplitude time series data of the exceptional VV bin reveals that the bin is much noisier than other measurements. Therefore, the generalized dimension spectrum D_q can accurately detect low observable targets within sea clutter.

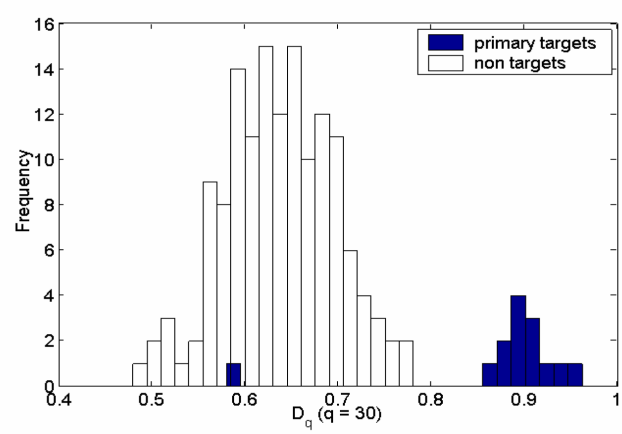

Fig.6 The PDFs of the bins without targets
and with primary targets for VV datasets

5. CONCLUDING REMARKS

In this paper, we introduce a multiplicative multifractal process, a convenient means for modeling sea clutter amplitude data. Through analysis of 392 amplitude time series of 14 sea clutter datasets measured under various sea and weather conditions, we show that the data are consistent with such multifractals over certain time scale range. Furthermore, we have found that the generalized dimension spectrum D_q can accurately detect low observable targets within sea clutter. The method thus may be very helpful in developing an automated target detector within sea clutter.

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